The Ellipsoidal Universe and the Hubble tension

Paolo Cea [1]

INFN - Sezione di Bari, Via Amendola 173 - 70126 Bari, Italy

Abstract

The Hubble tension resides in a statistically significant discrepancy between early time and late time determinations of the Hubble constant. We discuss the Hubble tension within the Ellipsoidal Universe cosmological model. We suggest that allowing small anisotropies in the large-scale spatial geometry could alleviate the tension. We, also, show that the discrepancy in the measurements of the Hubble constant is reduced to a statistically acceptable level if we assume sizeable cosmological anisotropies during the Dark Age. In addition, we argue that the Ellipsoidal Universe cosmological model should resolve the S_8 tension.

Keywords: Hubble constant, Cosmic Microwave Background

PACS: 98.80.Es, 98.70.Vc

[1]Electronic address: `paolo.cea@ba.infn.it`

One of the most fundamental cosmological parameter is the Hubble constant H_0 [1, 2] that measures the current expansion rate of the Universe. Recently, a statistically significant discrepancy has emerged between different methods of measuring the Hubble constant. Indeed, there is an evident tension between the Hubble parameter measured by late universe observations and the one measured by the Plank Collaboration (a fair exhaustive account can be found in the reviews Refs. [3, 4, 5] and references therein).
The Planck Cosmic Microwave Background (CMB) angular power spectra provided the most precise determination of the cosmological parameters. To extract H_0 from the CMB data it is necessary to assume a model for the expansion history of the Universe. Actually, the CMB measurements by the *Planck* satellite have confirmed to a high level of accuracy the standard Λ Cold Dark Matter (ΛCDM) cosmological model based on the flat Friedmann-Robertson-Walker metric:

$$ds^2 \;=\; -c^2\, dt^2 \;+\; a^2(t)\, \delta_{ij}\, dx^i\, dx^j \tag{1}$$

with a cold dark matter component and a dark energy component in the form of a cosmological constant. The 'TT, TE, EE + low E + lensing + BAO' best fit ΛCDM model to the Planck 2018 data [6] furnished for the Hubble constant:

$$H_0 \;=\; 66.76 \;\pm\; 0.42 \;\; km\; s^{-1}\; Mpc^{-1} \tag{2}$$

at the 68 % confidence level. On the other hand, the Supernovae H0 for the Equation of State (SH0ES) Team reported the most recent local measurement of H_0 obtained by the cosmic ladder of Cepheid-SN Ia standard candles [7]:

$$H_0 \;=\; 73.04 \;\pm\; 1.04 \;\; km\; s^{-1}\; Mpc^{-1} \quad . \tag{3}$$

These two independent estimates of the Hubble constant are in tension with each other at a significant statistical level reaching about five standard deviations.
In addition to the already mentioned Hubble constant disagreements, a tension between the Planck data with weak leasing measurements and the redshift surveys has been reported about the value of the parameter $S_8 = \sigma_8 \sqrt{\frac{\Omega_m}{0.3}}$, where Ω_m is the present time value of the nonrelativistic matter density and σ_8 is the amplitude of growth of structures. As a matter of fact, it is now well established that this S_8 tension is driven by σ_8 rather than Ω_m. To be concrete, here we report the recent measurement from a joint cosmological analysis of weak gravitational lensing observations from the Kilo-Degree Survey (KiDS-1000), with redshift-space galaxy clustering observations from the baryon Oscillation Spectroscopy Survey (BOSS) and galaxy-galaxy lensing observations. The combination between KiDS-1000, BOSS and the Spectroscopic 2-degree Field Lensing Survey (2dFLenS), presented in Ref. [8], resulted in the following constraint on the structure growth parameter:

$$S_8 \;=\; 0.766 \;^{+0.020}_{-0.014} \quad . \tag{4}$$

This valuye of the S_8 parameter should be compared with the one estimated by the Planck Collaboration within the standard ΛCDM cosmological model [6]:

$$S_8 \;=\; 0.825 \;\pm\; 0.011 \quad . \tag{5}$$

From Eq. (5) we see that there is a mismatch of about three standard deviations between the S_8 value estimated by the Planck Collaboration and the value reported in Eq. (4).

In the present note we perform an exploratory study of the cosmological tensions within the Ellipsoidal Universe cosmological model [9, 10] that was proposed to cope with several anomalous features at large scales in the cosmic microwave background anisotropy data. Indeed, even the Planck 2018 data confirmed the presence of large-scale anomalous features. As it is well known, the most evident anomaly concerned the quadrupole temperature correlation that was suppressed with respect to the best-fit ΛCDM cosmological model. In Refs. [9, 10] it was suggested that, if one allows the large-scale spatial geometry of the Universe to be only plane-symmetric, then the quadrupole amplitude can be drastically reduced without affecting the higher multipole correlations of the angular power spectrum of the temperature anisotropies. In the Ellipsoidal Universe the Friedmann-Robertson-Walker metric Eq. (1) is replaced by:

$$ds^2 \;=\; -c^2\, dt^2 \;+\; a^2(t)\;\left(\delta_{ij} \;+\; h_{ij}\right) dx^i\, dx^j \tag{6}$$

with

$$h_{ij}(t) \;=\; -\, e^2(t)\, n_i\, n_j \tag{7}$$

where $e(t)$ is the ellipticity, and the unit vector $\vec{n}$ determines the direction of the symmetry axis. Moreover, at variance with the standard cosmological model, the Ellipsoidal Universe model is able to account for large-scale CMB polarization without invoking reionization processes. Indeed, in our previous papers [11, 12, 13] we were able to fix the eccentricity at decoupling and the polara angles θ_n, ϕ_n of the direction of the axis of symmetry $\vec{n}$ such that the quadrupole temperature-temperature correlation matched exactly the Planck 2018 value. We found [13]:

$$e_{dec} \;=\; 8.32 \;\pm\; 1.32\;\; 10^{-3} \quad , \tag{8}$$

$$\theta_n \;\simeq\; 73^\circ \quad , \quad \phi_n \;\simeq 264^\circ \quad . \tag{9}$$

We, also, showed that the quadrupole TE and EE correlations compared reasonably well to the Planck 2018 data. These results allowed us to reach the conclusion that the Ellipsoidal Universe cosmological model not only were a viable alternative to the standard cosmological model, but also it seemed to compare observations better than the Λ Cold Dark Matter cosmological model.
We address, now, the problem to see if the anisotropies in the universe spatial geometry can be able to alleviate the Hubble and S_8 tensions. The Hubble constant can be inferred from the angular size of the sound horizon at recombination θ^* that, in turns, is given by the ratio of the comoving sound horizon to the comoving angular diameter distance to the last-scattering surface:

$$\theta^* \;=\; \frac{r_s(z^*)}{D_M(z^*)} \quad , \tag{10}$$

z^* being the redshift when the CMB radiation was last scattered. The comoving linear size of the sound horizon and the angular distance are linked to the expansion history of the Universe through:

$$r_s(z) \;=\; \int_z^\infty \frac{c_s(z')}{H(z')}\, dz' \tag{11}$$

and

$$D_M(z) \;=\; \int_0^z \frac{c}{H(z')}\, dz' \tag{12}$$

with $c_s(z)$ the speed of sound and $H(z)$ the Hubble constant at redshift z. At early times, relevant for computing the sound horizon at recombination, in the ΛCDM model one can write:

$$H(z) \simeq H_0 \sqrt{\Omega_\Lambda + \Omega_m (1+z)^3} \ , \tag{13}$$

where Ω_m and Ω_Λ are the fractional densities of matter and dark energy satisfying the constraint:

$$\Omega_\Lambda + \Omega_m = 1 \ . \tag{14}$$

So that we can rewrite Eq. (10) as:

$$\frac{c}{H_0} \int_0^{z^*} \frac{dz}{\sqrt{\Omega_\Lambda + \Omega_m (1+z)^3}} = \frac{r_s(z^*)}{\theta^*} \ . \tag{15}$$

Using the following values taken from Table 2 in Ref. [6]:

$$\Omega_\Lambda \simeq 0.689 \ , \ \Omega_m \simeq 0.311 \tag{16}$$

and

$$z^* \simeq 1090 \ , \ \theta^* \simeq 1.041 \times 10^{-2} \ , \ r_s(z^*) \simeq 144.6 \, Mpc \ , \tag{17}$$

we readily obtain from Eq. (15):

$$H_0 \simeq 67.9 \ \ Km \, s^{-1} \, Mpc^{-1} \tag{18}$$

that agrees with the best-fitted Hubble constant Eq. (2). Now, let us focus on the Ellipsoidal Universe model. In this case H(z) becomes:

$$H(z) \simeq H_0 \sqrt{\Omega_\Lambda + \Omega_m (1+z)^3 + \Omega_a(z)} \tag{19}$$

since the source of anisotropy adds the term $\Omega_a(z)$ due to a generic and unspecified anisotropic component related to the cosmic shear [14, 15]. Obviously, we have the constraint:

$$\Omega_\Lambda + \Omega_m + \Omega_a(0) = 1 \ . \tag{20}$$

It turned out [14, 15] that the cosmic shear is always smaller than unity. Moreover, the actual fraction of energy associated to the anisotropic component is negligible with respect to those of matter and dark energy. Therefore, to a good approximation we will assume in what follows:

$$\Omega_a(z) = 0 \ . \tag{21}$$

Nevertheless, it is worthwhile to mention the recent study presented in Ref. [16]. The authors of Ref. [16] considered an anisotropic generalization of the base ΛCDM model where the cosmic shear was assumed to behave like a stiff fluid. Interestingly enough, they found that even with a tiny source of anisotropy the mean value of H_0 and Ω_m are systematically larger than those in the case of the standard ΛCDM model, though with a rather low statistical significance. Thus, we see that the results of Ref. [16] are a first indication that a small anisotropy in the universe expansion rate tends to alleviate the H_0 tension. In addition, we need also to take into account the cosmological aberration that affect the measurement of the angular size of the sound horizon. To see this, we consider the null geodesic in the Ellipsoidal Universe:

$$c^2 \, dt^2 = a^2(t) \ \left(\delta_{ij} - e^2(t) \, n_i \, n_j\right) dx^i \, dx^j \ . \tag{22}$$

From this last equation, after averaging over the spatial directions, we infer:

$$c^2 \, dt^2 \;=\; a^2(t) \, \left[1 \;-\; \frac{1}{4} \, (1 + \frac{1}{3} \cos^2 \theta_n - \frac{1}{3} \sin^2 \theta_n \sin^2 \phi_n) \; e^2(t) \right] d\vec{x}^2 \;\; . \tag{23}$$

So that the comoving angular distance becomes:

$$D_M^{El}(z) \;=\; D_M(z) \;+\; \delta D_M(z) \tag{24}$$

with

$$\delta D_M(z) \simeq - \, \frac{1}{8} \, (1 + \frac{1}{3} \cos^2 \theta_n - \frac{1}{3} \sin^2 \theta_n \sin^2 \phi_n) \, \frac{c}{H_0} \int_0^z \frac{e^2(z')}{\sqrt{\Omega_\Lambda \;+\; \Omega_m \, (1 \;+\; z')^3}} \, dz' \; . \tag{25}$$

Now, let us suppose that θ^* is the comoving angular diameter distance to the last scattering surface. Thus, we get:

$$D_M^{El}(z^*) \, \theta^* \;=\; r_s(z^*) \;\; . \tag{26}$$

Using Eq. (24) we rewrite Eq. (26) as:

$$D_M(z^*) \, \theta^* (1 \;-\; \delta) \;=\; r_s(z^*) \;\; , \tag{27}$$

where:

$$\delta \;=\; - \, \frac{\delta D_M(z^*)}{D_M(z^*)} \;\; . \tag{28}$$

Equation (27) tells us that the measured comoving angular diameter assuming an isotropic spatial metric is:

$$\theta^*_{meas} \;\simeq\; \theta^* (1 \;-\; \delta) \;\; . \tag{29}$$

Therefore we can write:

$$D_M(z^*) \;\simeq\; \frac{r_s(z^*)}{\theta^*_{meas} (1 \;+\; \delta)} \;\; , \tag{30}$$

where θ^*_{meas} is given by Eq. (17). Combining Eqs. (25) and (28) it is easy to check that $\delta > 0$, so that Eq. (30) results in an estimate of the Hubble constant H_0 greater than that of the standard ΛCDM cosmological model. In the Ellipsoidal Universe model, according to Ref. [12] we can write (see Fig. 1):

$$e^2(z) \;\simeq\; e^2_{dec} \, \left[\frac{1 + z}{1 + z^*} \right]^{\frac{3}{2}} \;\; , \;\; z \;\leq\; z^* \;\; . \tag{31}$$

We obtain, then:

$$\delta \;\simeq\; 2.1 \;\times\; 10^{-7} \tag{32}$$

that, obviously, is too small to account for the H_0 tension. It is noteworthy that we may resolve the Hubble tension if we assume a finite period of sizeable anisotropies during the Dark Age, namely the period of time between the last scattering of the CMB radiation by the almost homogeneous plasma and the formation of the first star, Indeed, if we assume (see Fig. 1):

$$e^2(z) \;\simeq\; 0.90 \;\; , \;\; z_1 \;\simeq\; 15 \;\leq\; z \;\leq\; z_2 \;\simeq\; 300 \;\; , \tag{33}$$

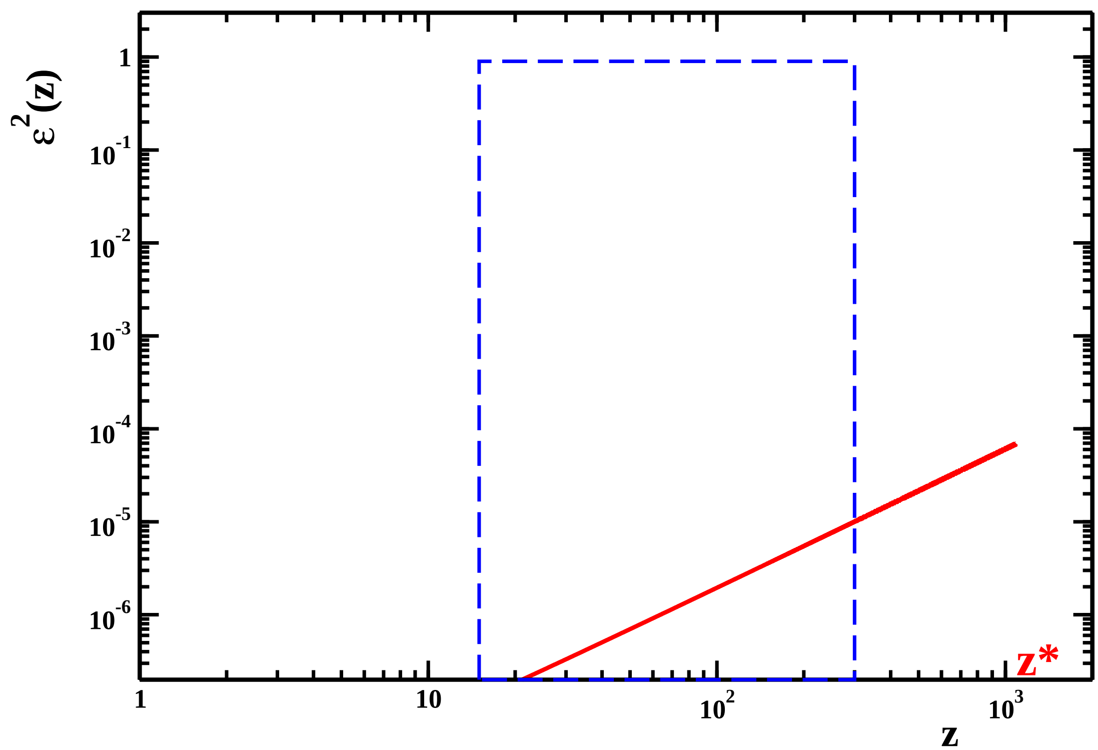

Figure 1: Ellipticity as a function of the redshift z. The (red) continuous line corresponds to Eq. (31), the (blue) dashed line to Eq. (33).

we obtain:

$$\delta \simeq 3.4 \times 10^{-2} . \tag{34}$$

After using Eq. (30), one gets:

$$H_0 \simeq 70.3 \;\; Km\, s^{-1}\, Mpc^{-1} \;\; , \tag{35}$$

that agrees with Eq. (3) within about two standard deviations. Note that the cosmic shear generated during such an extended period of time does not give rise to additional temperature anisotropies since the integrated Sachs-Wolfe effect vanishes:

$$\frac{\delta T}{T} \simeq - \frac{1}{2} \int_{t^*}^{t_0} dt \, \frac{\partial h_{ij}(t)}{\partial t} \, n^i \, n^j = 0 \tag{36}$$

where t_0 is the present time (z=0). Therefore, presumably, the only effects left should be some anisotropies in the matter distribution. In this respect, it is interesting to note that recent studies (see Refs. [17, 18, 19, 20] and references therein) based on observations of quasars, supernovae and gamma ray bursts provided some evidences for an anisotropic departure from the Friedmann-Robertson-Walker metric. However, it is difficult to image physical processes able to generate sizeable anisotropies in early times. One might think of cosmic defects drawn across space that crossed through the Universe in the Dark Ages. Even thought such a mundane possibility is logically admissible, it should be evident that the Ellipsoidal Universe cosmological model can accomodate values of H_0 larger than in the standard ΛCDM model, whilst not degrading the fits to the CMB data. After all the Ellipsoidal Universe model amounts to a simple anisotropic correction to the standard

cosmological model by replacing the spatially flat metric with the plane-symmetric Bianchi type-I metric. In this way one introduces additional cosmological parameters that should be best-fitted to the precise Planck measurements. Our previous discussion illustrated how a tiny variation of the ΛCDM parameters resulted in an appreciable relaxation of the Hubble tension. This last point can be better appreciated by looking at the S_8 tension.
As we said before, the S_8 tension arises from measurements with weak-lensing Planck data and redshift surveys. We, also, noticed that the tension is mainly driven by σ_8. On the other hand, the amplitude of density perturbation σ_8 is tightly related to the primordial comoving curvature power spectrum amplitude A_s defined, conventionally, at the pivot scale $k_{pivot} = 0.05\,Mpc^{-1}$. The CMB lensing reconstruction power spectrum constrains the late-time fluctuation amplitude more directly in combination with matter density. Therefore, the dependence of the lensing power spectrum on A_s can be eliminated in favour of σ_8. The parameter dependence is given by [21]:

$$\sigma_8^2 \;\propto\; A_s\,\Omega_m^{1.5}\,h^{3.5} \;, \tag{37}$$

where $H_0 = 100 \times h\;Km\,s^{-1}\,Mpc^{-1}$.
The observed CMB power spectrum amplitude scales with the primordial comoving curvature spectrum A_s. Actually, the observed amplitude scales with $A_s\,\exp(-2\tau)$ (τ being the optical depth) due to the scatterings of free electrons that are present after reionization. Therefore, it is the combination $A_s\,\exp(-2\tau)$ that is well measured [6]:

$$A_s\;\exp(-2\tau) \;=\; 1.881 \,\pm\, 0.010 \,\times\, 10^{-9} \;. \tag{38}$$

In the standard ΛCDM cosmological model it is assumed that the large-scale CMB polarization is due to reionization processes. Thus, low-ℓ E-mode polarization powers are dominantly produced by Thompson scattering of CMB photons off the free electrons which are produced by reionization. So that the optical depth and the reionization redshift z_{re} are well constrained by the large-scale polarization measurements [6]:

$$z_{re} \;=\; 7.82 \,\pm\, 0.71 \;\;,\;\; \tau \;=\; 0.0561 \,\pm\, 0.0071 \;. \tag{39}$$

Combining Eqs. (38) and (39) one gets:

$$A_s \;=\; 2.105 \,\pm\, 0.030 \,\times\, 10^{-9} \;. \tag{40}$$

In the Ellipsoidal Universe model we showed [11, 12, 13] that there is sizeable large-scale polarization signal without invoking reionization processes. Moreover, we found [13] the CMB quadrupole TE and EE correlations were in agreements with the Planck 2018 data. As a consequence, in the Ellipsoidal Universe cosmological model the optical depth is not constrained, but it must be much smaller than the best-fit value Eq. (39). Assuming $\tau \simeq 0$, from Eq. (38) we estimate:

$$A_s^{El} \;\simeq\; 1.881 \,\times\, 10^{-9} \;, \tag{41}$$

that is smaller with respect to the standard cosmological model, Eq. (40). This, in turns, reduces the amplitude of density perturbation via Eq. (37) and leads to:

$$S_8^{El} \;\simeq\; 0.780 \tag{42}$$

that seems to be close enough to Eq. (4) so as to eliminate the S_8 tension.

The Ellipsoidal Universe cosmological model amounts to a small anisotropic correction to the base ΛCDM cosmological model, that was proposed several years ago [9, 10] to explain the CMB quadrupole anomaly. Since then, we have shown [11, 12, 13] that the Ellipsoidal Universe proposal can, also, produce large-scale CMB E-mode correlations in agreement with the latest Planck data. In the present note we are suggesting that the Ellipsoidal Universe cosmological model should alleviate both the Hubble and S_8 tensions.

References

[1] N. Jackson, The Hubble Constant, Living Rev. Rel. **10** (2007) 4

[2] W. L. Freedman and B. F. Madore, The Hubble Constant, Ann. Rev. Astron. Astrophys. **48** (2010) 673

[3] L. Knox and B. F. Madore, Hubble constant hunter's guide, Phys. Rev. D **101** (2020) 043533

[4] P. Shah, P. Lemos and O. Lahav, A buyer's guide to the Hubble constant, Astron. Astrophys. Rev. **29** (2021) 9

[5] E. Di Valentino, *et al.*, In the realm of the Hubble tension - a review of solutions, Class. Quantum Grav. **38** (2021) 153001

[6] N. Aghanim, *et al.*, Planck Collaboration, Planck 2018 results. VI. Cosmological parameters, Astron. Astrophys. **641** (2021) A6

[7] A. G. Riess, *et al.*, A Comprehensive Measurement of the Local Value of the Hubble constant with 1 km s^{-1} Mps^{-1} Uncertainty from the *Hubble Space Telescope* and the SH0ES Team, arXiv:2112.04510 [astro-ph.CO] (2021)

[8] C. Heymans, *et al.*, KiDS-1000 Cosmology: Multi-probe weak gravitational lensing and spectroscopic galaxy clustering constraints, Astron. Astrophys. **646** (2021) A140

[9] L. Campanelli, P. Cea and L. Tedesco, Ellipsoidal Universe Can Solve the Cosmic Microwave Background Quadrupole Problem, Phys. Rev. Lett. **97** (2006) 131302

[10] L. Campanelli, P. Cea and L. Tedesco, Cosmic microwave background quadrupole and ellipsoidal universe, Phys. Rev. D **76** (2007) 063007

[11] P. Cea, On the large-scale cosmic microwave background polarization, Mon. Not. R. Astron. Soc. **406** (2010) 586

[12] P. Cea, The ellipsoidal universe in the *Planck* satellite era, Mon. Not. R. Astron. Soc. **441** (2014) 1646

[13] P. Cea, Confronting the ellipsoidal universe to the Planck 2018 data, Eur. Phys. J. Plus **135** (2020) 150

[14] L. Campanelli, P. Cea , G. L. Fogli and L. Tedesco, Cosmic Parallax in Ellipsoidal Universe, Mod. Phys. Lett A **26** (2011) 1169

[15] L. Campanelli, P. Cea, G. L. Fogli and L. Tedesco, Anisotropic dark energy and Ellipsoidal Universe, Int. J. Mod. Phys. D **20** (2011) 1153

[16] Ö. Akarsu, S. Kumar, S. Sharma and L. Tedesco, Constraints on a Bianchi type I spacetime extension of the standard ΛCDM model, Phys. Rev. D **100** (2019) 023532

[17] K. Migkas, G. Schellenberger, T. H. Reiprich, F. Pacaud, M. E. Ramos-Ceja and L. Lovisari, Probing cosmic isotropy with a new X-ray galaxy cluster sample through the $L_X - T$ scaling relation, Astron. Astrophys. **636** (2020) A15

[18] J. P. Hu, Y. Y. Wang and F. Y. Wang, Testing cosmic anisotropy with Pantheon sample and quasar at high redshifts, Astron. Astrophys. **643** (2020) A93

[19] C. Krishnan, R. Mohayaee, E. O Colgain, M.M. Scheikh-Jabbari and L. Yin, Hints of FLRW Breakdown from Supernovae, arXiv:2106.02532 [astro-ph.CO] (2021)

[20] O. Luongo, M. Muccino, E. O Colgain, M.M. Scheikh-Jabbari and L. Yin, On larger H_0 values in the CMB Dipole direction, arXiv:2108.13228 [astro-ph.CO] (2021)

[21] P. A. R. Ade, *et al.*, Planck Collaboration, Planck 2015 results. XV. Gravitational lensing, Astron. Astrophys. **594** (2016) A15