The **Λ-Cold-Dark-Matter Model**

Summary by: Louis Marmet, 16th April, 2019

The ΛCDM model is based on the interpretation that the observed redshift z of distant galaxies is a result of the expansion of space. Extrapolating backwards in time gives the prediction that the universe *expanded from an extremely dense and hot state*, the Big Bang, about 13.80 Gyr ago. The model explains the Hubble law, the Cosmic Microwave Background (CMB), the abundance of light elements and the large scale structure. Six parameters [displayed below in square brackets] are required to give a reasonable fit to observations.

After the Big Bang, the universe went through several phases:[1] the scalar spectral index [$n_s \approx 0.967$] and the curvature fluctuation amplitude [$\Delta_R^2 \approx 2.44$] characterize a rapid inflation which amplified quantum fluctuations to produce the large-scale structure; a baryon-number violation produced a small imbalance which favoured matter over anti-matter (baryogenesis); with decreasing temperatures, quarks formed protons and neutrons; photons became the dominating particles; light elements were produced (nucleosynthesis); electrons and protons combined into hydrogen; matter and radiation decoupled, producing what would become the CMB; denser regions gravitationally attracted matter to form stars and galaxies; starlight produced reionization at an optical depth[2] [$\tau \approx 0.07$]; the expansion accelerated once dark energy started dominating its evolution.

The model is derived from general relativity with a cosmological constant to describe the universe between inflation and the present. It uses the Friedmann-Lemaître-Robertson-Walker metric, the Friedmann equations and the cosmological equations of state. The cosmic scale factor $a(t) = (1+z)^{-1} = (\Omega_m/\Omega_\Lambda)^{1/3} \sinh^{2/3}\left(\frac{3}{2}H_0\sqrt{\Omega_\Lambda}\, t\right)$ quantifies the Hubble law and $H(a) = H_0\sqrt{\Omega_m a^{-3} + \Omega_\Lambda}$ is the Hubble parameter with [$H_0 \approx 67.7$ km/s/Mpc]. The comoving distance[3] is $d_C = \dfrac{c}{H_0}\displaystyle\int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}}$, where: the equation of state of dark energy is $w \equiv -1$, density parameters are expressed as a dimensionless ratio Ω for various species and add up to $\Omega_{tot} \equiv 1$, the curvature $\Omega_k \equiv 0$, the radiation density (photons) $\Omega_r \sim 10^{-4}$ is neglected, Ω_m is the matter density (the sum of cold dark [$\Omega_c \approx 0.259$], baryonic [$\Omega_b \approx 0.049$], and a negligible neutrino density), and the cosmological constant $\Omega_\Lambda \sim 1 - \Omega_m \sim 0.691$.

The angular distance is

$$d_A = d_C/(1+z).$$

The luminosity distance[4]

$$d_L = (1+z)^2 d_A$$

results from luminosity reduction due to the redshift (factor $\sqrt{1+z}$), the lower rate at which the photons reach the observer (factor $\sqrt{1+z}$) and the reduced solid angle (factor $1+z$).

The distance modulus is[5]

$$\mu = 10\log_{10}(1+z) + 5\log_{10}(d_A/D_H) + C,$$

where C is an object-specific constant.

Due to galactic luminosity evolution, the observed surface brightness follows a $2.5\log(1+z)^{4-p}$ law with a wavelength dependent value $0.5 < p < 1.2$.[6]

The time dilation factor is a relativistic effect given by

$$\gamma = 1 + z = a^{-1}(t),$$

describing the longer wavelength of redshifted light and the stretched light curves of Type Ia Supernovae.

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[1] "Lambda-CDM model," Wikipedia en.wikipedia.org/wiki/Lambda-CDM_model#Parameters, and references within.

[2] "Optical Depth to Reionization, τ," NASA Lambda, lambda.gsfc.nasa.gov/education/graphic_history/taureionzation.cfm.

[3] D.W. Hogg, "Distance measures in cosmology," arXiv:astro-ph/9905116v4, May 1999.

[4] J.M. McKinley, "Relativistic transformations of light power," Am. J. Phys., 47:602 (1979), dx.doi.org/10.1119/1.11762.

[5] P.D. Mannheim, "Alternatives to Dark Matter and Dark Energy," arXiv:astro-ph/0505266, p. 45.

[6] A. Sandage, The Astronomical Journal, vol. 139, no. 2, p. 728, Feb. 2010.