

# Cosmology without a cosmological constant.

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## ABSTRACT

In order to deal with the coincidence problem a new scalefactor-redshift relation is considered. With the new relation a constant rate of expansion with no cosmological constant matches observations well. There is thus the possibility that the dark energy phenomenon is an illusion caused by the use of an incorrect relation. Collaboration is invited to see if a cosmology without a cosmological constant can be developed.

## INTRODUCTION

The LCDM model is currently favoured by the majority of cosmologists. The dark energy component, although lacking a theoretical understanding, has enabled the model to successfully match different sets of observations.

- 1) The distance moduli of supernovae (Betoule [1])
- 2) Measurements of the CMB, e.g. WMAP (Hinshaw [2]) and Planck (Ade [3]) appear to show a flat universe. The matter density parameter  $\Omega_m$  is about 0.25 to 0.31, deduced from measurements of  $\Omega_m h^2$ , (where  $h$  is the Hubble parameter/100km/s/Mpc).
- 3) There are other observations (Weinberg [4]). Many of these also depend on the value of  $h$  or  $\Omega_m h^2$
- 4) Baryon Acoustic Oscillations (Aubourg [5]), give a slightly lower value for  $h$  and a higher  $\Omega_m$ , although there are inconsistencies the distance ladder (Riess [6]) and between the Ly $\alpha$  and galaxy redshift samples (these are reconciled in Appendix C).

Observations 1-4 seem to support each other, so it is understandable that many cosmologists support the dark energy conclusion. However a simple change to our notions of how scale factor relates to redshift can remove many of the arguments in favour of the accelerating universe and dark energy.

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## 1. A NEW RELATIONSHIP BETWEEN REDSHIFT AND SCALE FACTOR

Traditionally the wavelength of light received is proportional to the ratio of the scale factor of the universe on emission and reception, giving the redshift of a photon

$$1 + z = \frac{\lambda_0}{\lambda_1} = \frac{a_0}{a_1} \quad (1)$$

with the new proposal there is a different relation

$$1 + z = \frac{\lambda_0}{\lambda_1} = \left( \frac{a_0}{a_1} \right)^2 \quad (2)$$

The general features of a new cosmological model are discussed in Appendix B, with a reason why (2) could be true and (1) is false. There may be other models too, so the consequences of relation (2) are the subject of the main part of this paper, how any model which has relation (2) could lead to the apparent dark energy phenomenon if cosmologists are mistakenly using (1).

The rate of expansion of the universe

$$H(t) = \frac{da/dt}{a} \quad (3)$$

If relation (2) is true

$$1 + z = (1 + Hdt)^2 = 1 + 2Hdt \quad \text{for small } Hdt, \text{ so}$$

$$v = 2Hd \quad (4)$$

equation (4) relates the apparent velocity of a source to distance. So Hubbles constant  $H_0$ , is twice the rate of expansion  $H(t)$ , assumed constant, hereafter  $H$ .

$$H = \frac{H_0}{2} \quad (5)$$

## 2. THE MATTER DENSITY

For a simple constant  $H$  solution,  $a = a_0 \exp(Ht)$ , Einsteins equations of General Relativity reduce to

$$8\pi G \frac{\rho}{c^2} = -\Lambda + \frac{3H^2}{c^2} + \frac{3k}{a^2} \quad (6)$$

$$8\pi G \frac{p}{c^4} = \Lambda - \frac{3H^2}{c^2} - \frac{k}{a^2} \quad (7)$$

so for a flat universe with  $k = 0$ , and  $\Lambda = 0$

$$p = -c^2 \rho \quad (\text{i.e. } \omega = -1) \quad (8)$$

and

$$\rho = \frac{3H^2}{8\pi G} \quad \text{or} \quad G = \frac{3H^2}{8\pi\rho} \quad (9)$$

for this, and other solutions, the inferred value for the matter density would be

$$\Omega_m = \frac{\rho}{\rho_{crit}} = \frac{3H^2/8\pi G}{3(H_0)^2/8\pi G} = 0.25 \quad (10)$$

whereas actually  $\Omega_m$  is 1, as the denominator of (10) should contain  $H$  not  $H_0$ .

Measurements from WMAP9 alone, led to an inferred value for the matter density parameter of  $0.279 \pm 0.025$ . This value is deduced from measurements of  $\Omega_m h^2$  and  $\Omega_b h^2$  with, possibly, an incorrect value for  $h$ , and would be  $1.12 \pm 0.1$  with  $h$  halved.

In this example the ‘coincidence problem’ is avoided. At all times the matter density is 1 and the cosmological constant is 0.

### 3. THE SUPERNOVAE DATA

With the new approach  $H$  is about 35km/s/Mpc and the comoving distance is

$$d_c = \frac{c}{H} (\sqrt{1+z} - 1) \quad (11)$$

(there is a derivation in Appendix A), so

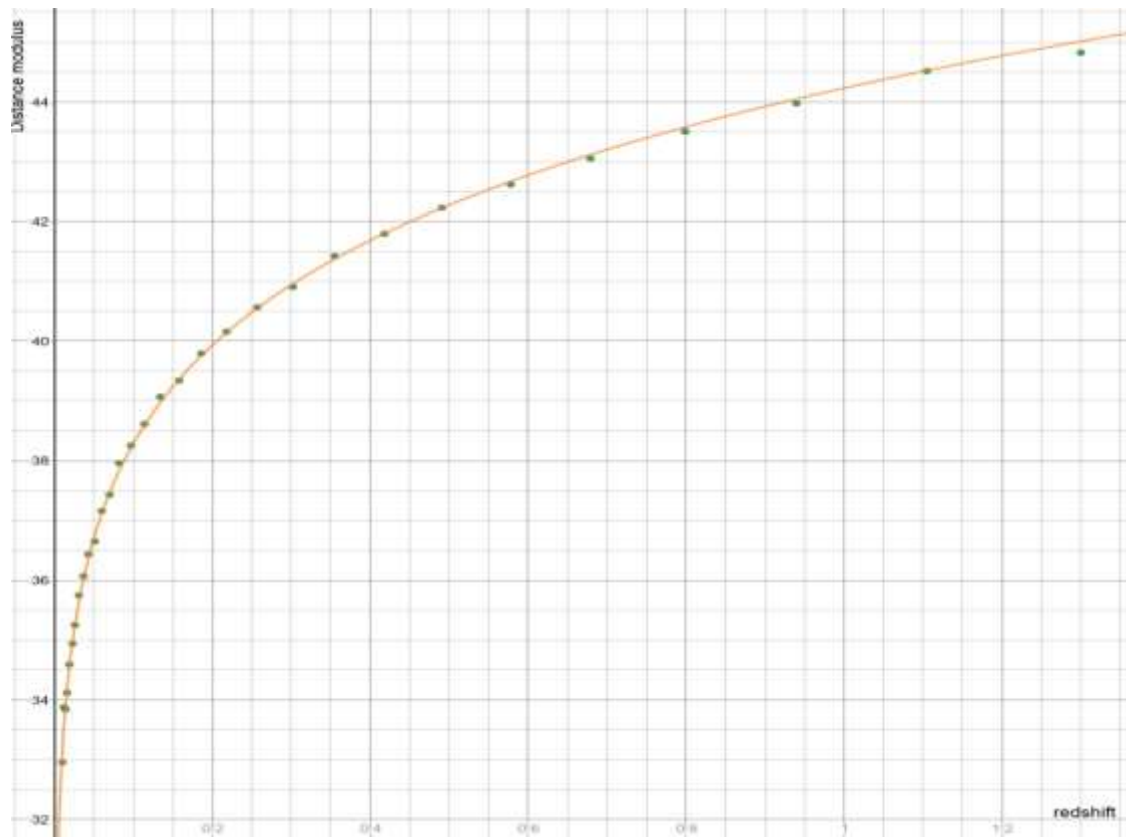
$$d_L = \frac{c}{H} (1+z) (\sqrt{1+z} - 1) \quad (12)$$

the distance modulus is

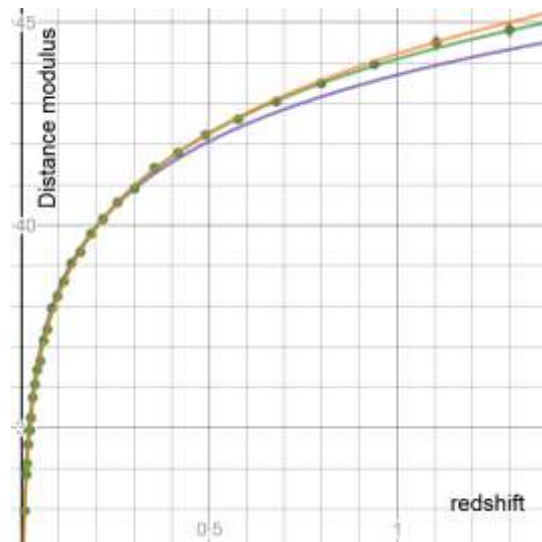
$$\mu = 25 + 5 \log d_L \quad (13)$$

Using (12) in (13), there is a good match to supernovae data. The Betoule binned data, is shown in Figure 1, with the new relation. The best fit is with  $2H$  constant, 70.9 km/s/Mpc, and no other variable parameters.

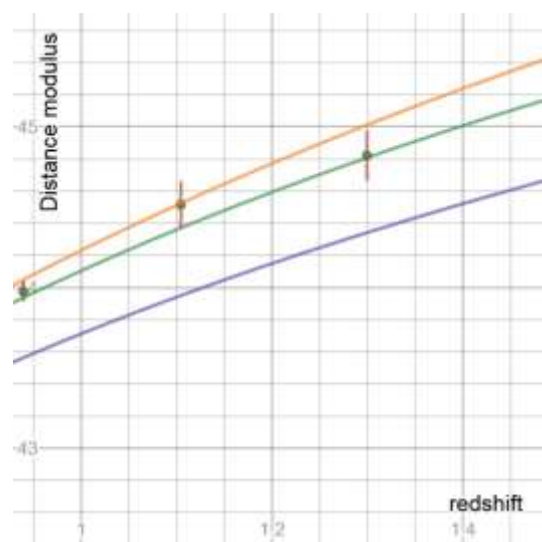
Figure 1a Distance modulus against redshift, (new relation, constant H)



1b New relation, LCDM  $\Omega_m = 0.3$  and 1.0



1c Enlargement of 1b



ΛCDM also gives a close match, middle curve 1b and c, but with two variable parameters,  $h$  and  $\Omega_m$ .

The binomial expansions for the ΛCDM and the new relation are now compared. For ΛCDM,  $E(z)=[m(1+z)^3+1-m]^{0.5}$ , where  $m$  is short for  $\Omega_m$

$$d_c = \frac{c}{H_0} \int_0^z \frac{dz}{E(z)} \quad (14)$$

i.e.

$$d_c = \frac{c}{H_0} \left( z - \frac{3mz^2}{4} + \dots \right) \quad (15)$$

Whereas for (11) the binomial expansion is

$$d_c = \frac{c}{H_0} \left( z - \frac{z^2}{4} + \dots \right) \quad (16)$$

So for low  $z$  a match would occur if  $\Omega_m = 1/3$  in ΛCDM. For  $z=1$  the match occurs at 0.19, (using equal  $H$ ). Most of the data is at low redshift, so if ΛCDM is trying to match formula (12), by varying  $\Omega_m$ , we would expect it to be slightly lower than 1/3.

#### 4. CONCLUSIONS AND PREDICTIONS

There may have been a long-held misunderstanding of the relationship between the scale factor of the universe and redshift. The relation may be (2). If this is the case, we would expect the following.

- i) The inferred value of  $\Omega_m$  will be 0.25 from CMB data alone.
- ii) The distance moduli (13) of supernovae, will be according to (12), and the inferred value of  $\Omega_m$  will be slightly lower than 1/3.

The cosmological constant may be 0 and the expansion of the universe may not be accelerating.

#### ACKNOWLEDGEMENTS

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## APPENDIX A: DERIVATION OF (12)

Starting from the Robertson-Walker metric

$$ds^2 = -c^2 dt^2 + a^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad (\text{A1})$$

in terms of the co-moving co-ordinates,  $\chi$  has the role of the radial co-ordinate

$$\begin{aligned} r &= \sin \chi & \text{if } k &= +1 \\ r &= \chi & \text{if } k &= 0 \\ r &= \sinh \chi & \text{if } k &= -1 \end{aligned}$$

For ray of light moving along a radial path with  $\theta$  and  $\phi$  constant, for a flat universe,

$$ds^2 = -c^2 dt^2 + a(t)^2 d\chi^2 = 0 \quad (\text{A2})$$

so

$$ad\chi = -cdt = -c \frac{dt}{da} da = -c \frac{da}{\dot{a}} = -c \frac{da}{aH} \quad (\text{A3})$$

for the new relation (2)

$$1+z = \left[ \frac{a_0}{a_t} \right]^2 \quad (\text{A4})$$

$$dz = -2 \frac{a_0^2}{a^3} da \quad (\text{A5})$$

so from (A3)

$$ad\chi = \frac{c}{2H} \frac{a^2}{a_0^2} dz \quad (\text{A6})$$

$$a_0 d\chi = \frac{c}{2H} \frac{a}{a_0} dz \quad (\text{A7})$$

from (2)

$$a_0 d\chi = \frac{c}{2H} \frac{1}{\sqrt{1+z}} dz \quad (\text{A8})$$

$$d_c = \int_0^z \frac{c}{2H} (1+z)^{-\frac{1}{2}} dz \quad (\text{A9})$$

$$d_c = \frac{c}{H} (\sqrt{1+z} - 1) \quad (\text{A10})$$

which is (11), with  $H$  about 35km/s/Mpc and

$$d_L = \frac{c}{H} (1+z) (\sqrt{1+z} - 1) \quad (\text{A11})$$

which is (12).

## APPENDIX B: RESCALING SYMMETRY, THE ORIGIN OF GRAVITATION.

### B1 Introduction.

A new cosmological model is presented based on a symmetry principle. It provides an understanding of the origin of gravitation and the apparently accelerating universe phenomenon. It is necessarily incomplete and there will be many unanswered questions. Collaboration is invited by more mathematically minded physicists and cosmologists to see if a way forward can be found along these lines, and a cosmology developed without a cosmological constant.

Newtons Law of gravitation provides a description of the forces between masses. Einstein, with General Relativity, provided a more accurate description from how masses curve space-time. However neither of these theories deal with the question of why gravity exists.

Why does the attractive force depend on the size of a mass, or why does a larger mass curve spacetime more than a smaller mass? Why does gravitation exist at all? Why does  $G$  have its value? All these are unanswered.

A symmetry principle can help with these questions.

Einstein found that his equations did not allow a static universe. The universe must be expanding or contracting. Einsteins equations do not specify that only distances between galaxies are expanding. The expansion is not limited in principle to cosmological distances. If the expansion is for material objects and masses as well as distances between the masses, we find a simple and satisfying reason for the existence of gravitation, discussed in B4.

There now follows a fundamental reinterpretation of what Einstein's equations are telling us. Instead of the rate of expansion of the universe,  $H$ , being changed by gravitation – it is the expansion which causes gravitation. Gravitation is the result of the expansion hereafter called 'rescaling' and a symmetry principle, the rescaling symmetry principle.

The rescaling is of rate  $H$ , constant and unaffected by gravitation or any other force. It exists, postulated as a fundamental physical reality with  $H$  being a fundamental physical constant.

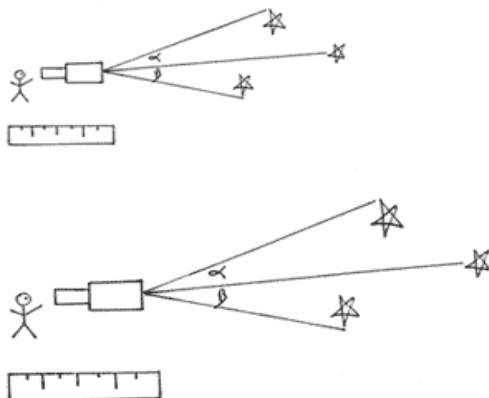
## B2 The rescaling symmetry principle.

According to the rescaling symmetry principle, if every length in the universe were to increase or decrease in proportion there would be no noticeable effect to the inhabitants, (figure B1).

As can be seen in the cartoon, angles and measured distances are unaffected.

This continuous and ongoing change in length scale must happen to every length in the whole universe simultaneously, including the size of people, atoms and distances between all objects. Every physical constant must vary too, with the change depending on the number of length dimensions in the quantity.

Figure B1 Sketch to show a rescaling universe





A common cosmological time ( $t$ ) is assumed.

Quantities rescale according to

$$\frac{dQ}{Q} = nHdt \quad (\text{B1})$$

where ‘ $n$ ’ is the number of length dimensions in quantity  $Q$ .  $H$  is the rescaling constant, which is half of Hubbles constant  $H_0$

$$Q = Q_0 \exp(nHt) \quad (\text{B2})$$

Table B1. The value of ‘ $n$ ’ for various physical quantities.

Quantity	$n$
all lengths	1
speed of light	1
Plancks constant	2
particle masses	0
permittivity of free space	-3
scalefactor of universe ‘ $a$ ’	1
gravitational constant	3
Hubbles constant	0
Forces	1
quanity with $n$ length dimensions	$n$

With this system the symmetry principle requires that any local experiment, to measure the change of any physical quantity, in a rescaling universe, would yield a null result. This is due to other relevant quantities rescaling too.

For example, imagine if an attempt were made to measure the change in the speed of light by timing the passage of a light beam over a given distance, 10 years apart. Since both the distance and the speed of light rescale in proportion the time of passage would remain the same.

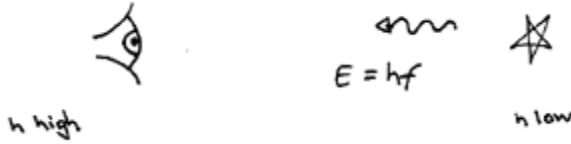
It could be argued that it’s meaningless to consider such a changing of scale – that because it’s immeasurable it cannot be happening. As the rescaling symmetry principle applies to the whole universe simultaneously it could also be argued that the universe could be regarded as static, with no change of any physical quantity. However there is an important difference. In a rescaling universe (assumed expanding) the universe and various physical constants are larger now than they used to be.

This leads to some observational differences between the static and rescaling universe cases. These arise from the conservation of energy, as described below.

## B2 The redshift of light.

In a rescaling universe, a photon of light arriving from a distant star, would be emitted at a time when Plancks constant was lower.

Figure B2 The redshift of light



By the time it has arrived at earth Plancks constant would be

$$h_0 = h_1 \exp(2Ht) \quad (\text{B3})$$

where  $t$  is the travel time of the photon.

If the energy of the photon is conserved

$$f_0 = f_1 \exp(-2Ht) \quad (\text{B4})$$

So light received from a distant source becomes redshifted. In this model the redshift of light is due to the rescaling universe, instead of an expanding universe.

The redshift of light is from

$$1 + z = \frac{\lambda_0}{\lambda_1} = \exp(2Ht) = \left( \frac{a_0}{a_1} \right)^2 \quad (\text{B5}) \quad \text{also (2)}$$

every quantity  $Q$  with  $n$  length dimensions, at redshift  $z$ , has the relation

$$Q(z) = Q_0 / (1 + z)^{n/2} \quad (\text{B6})$$

As we can only observe angles, timings and photons of light arriving from distant sources it is not easy to verify the rescaling this way.

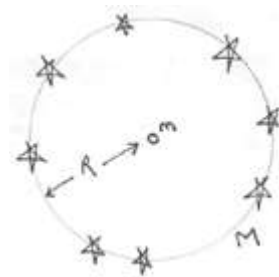
The error made in the derivation of (1) found in textbooks is that the expansion has been applied only to cosmological distances, not universally to all distances. With the traditional redshift-scalefactor relation, the photon loses energy as it travels, whereas in the above the energy of the photon is conserved.

The energy of each mass  $m$  is also conserved during the rescaling, due to gravitation as discussed below.

#### B4 The origin of gravitation

Newtonian arguments are used. Consider a mass  $m$  in a rescaling universe, by its presence the mass generates gravitational potential energy – but why?

Figure B3 A small mass  $m$  and the surrounding universe



The total energy due to each mass  $m$  is

$$mc^2 - \frac{GMm}{R} \quad (B7)$$

at a later time the total energy would be

$$(mc^2 - \frac{GMm}{R})\exp(2Ht) \quad (B8)$$

the second term in (B7) represents the combined contributions to the potential energy, due to  $m$ , of the rest of the universe, of mass  $M$ , up to the Hubble radius  $R$ . If energy is conserved in a rescaling universe the term (B7) must be zero

$$G = \frac{Rc^2}{M} \quad (B9)$$

Small numerical constants are omitted for simplicity.

The significance of equation (B9) is that gravity is caused by rescaling – i.e. the phenomenon of gravitation and the value of  $G$ , is a result of the conservation of energy in a rescaling universe.

This naturally leads to a universe near critical density, and a natural solution of the ‘flatness problem’. An exact treatment would need General Relativity and others are invited to attempt it. The equivalent of (B9) is (9).

There is a reduction in the value of  $G$  for masses of high mass to radius ratio.

For a large stationary mass, (B7) is amended to

$$mc^2 - \frac{GMm}{R} - \frac{Gm^2}{r} = 0 \quad (B10)$$

giving

$$G_{effective} = \frac{c^2}{(M/R + m/r)} \quad (B11)$$

so, from (B7)

$$G_{effective} = \frac{c^2}{\left(\frac{c^2}{G} + \frac{m}{r}\right)} \quad (B12)$$

Equation (B12) indicates that a version of General Relativity which incorporates the rescaling symmetry principle will have an effective value of  $G$  (or active gravitational mass of a body), will varies from object to object. For masses of extremely high  $m/r$  ratio,  $G$  will decrease.

Whilst the value of  $G$  is constant within General Relativity, a future understanding of the theory or an amended version of it, may incorporate this feature. Some scalar tensor theories (Brans [7]) have a variable gravitational constant. They have apparently been ruled out by Lunar Laser Ranging (LLR). If the rescaling interpretation is valid, such theories may merit further consideration as the change in  $R$  and the variation in  $G$  may not be a measurable one.

Equation (B12) predicts an annual variation in Earth's gravity, as the earth-sun distance changes, of  $3.3 \times 10^{-10}G$ . This variation may already have been detected. Measurements from Satellite Laser Ranging (SLR) data (Matsuo [8]), show a variation in geoid height of about that magnitude, although the cause is uncertain. The annual variation of the earth-moon distance is much larger due to other factors.

Such a mechanism allows large collapsing masses to 'bounce' giving rise to explosive, or ejection phenomenon on various scales. A future theory may be able to account for the foam-like large scale structure, ejections from AGNs and incorporate the successes of Big Bang cosmology.

If and when the nature of dark matter is understood, there may remain the outstanding question of why it is distributed in such a way as to give the flat rotation curves (Zwicky [9]). The distribution of dark matter may be determined by (B12). Matter approaching a galactic centre could only spiral in at such a rate, so as to give a constant  $m/r$  ratio for every value of  $r$ . If matter approached faster, the value of  $G(m, r)$  would be reduced, allowing matter to drift away from the centre, reducing the  $m/r$  ratio. A constant  $m/r$  ratio is thus maintained, this leads to the constant velocity of rotation. Incoming matter being periodically ejected perpendicular to the disk.

## APPENDIX C: RECONCILIATION OF VALUES OF HUBBLES CONSTANT.

There is currently a tension between the Hubbles constant values from the distance ladder approach (Riess[6]) and the Planck consortium (Ade[3]).

Consistency can be found from Riess and Planck data as follows.

1) Planck uses BAO data to determine  $H_0$ . Section 5.2 formula (25), has the expression  $\frac{cz}{H}$ , in which should be  $\frac{2c(\sqrt{1+z})-1}{H}$   
A  $z$  of 0.57 is used, so BAO might overestimate distances by a factor 1.1265 (0.57 compared to 0.505993) and hence underestimate  $H_0$  by the same factor. Plancks value of 67.3km/s/Mpc, formula (27), multiplied by 1.1265 becomes 75.8 km/s/Mpc.

2) With  $\Omega_m$  as 0.25 and using the Planck value for  $\Omega_m h^2$  of 0.1428 gives  $H(0) = 75.6\text{km/s/Mpc}$ .

3) Riess et al has an  $H_0$  of  $73.24 \pm 1.74$  km/s/Mpc, but in the analysis uses a deceleration parameter  $q_0$  derived from his 2007 paper, which uses the assumption of the LCDM model. The  $q_0$  should be -1. When this value is applied to their equation (5) it changes  $a_x$  and so from equation (4) changes  $H_0$ , by a factor  $10^{a_x}$ . The data has redshifts between 0.023 and 0.15, with more data near the 0.15 side. The required change is: redshift 0.023 it's by a factor 1.005, for  $z=0.05$  it's 1.011, for  $z=0.1$  it's 1.0235 and for  $z=0.15$  it's 1.036 so if a reasonable factor of 1.025 is applied we find a corrected value of  $73.24 \times 1.025 = 75.1 \pm 1.78$  km/s/Mpc

1-3) are consistent with each other given the errors involved and have an average value of about 75.5km/s/Mpc.