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Theory of the Redshift of the Spectral Lines of Cosmological Objects

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To interpret the peculiarities of the redshifts of the spectral ıes of light coming from distant cosmic objects, the author a previous report put forward a hypothesis according to hich cosmological space conducts electricity and thus is able absorb electromagnetic radiation which passes through it. In present report the author gives this theory a deeper eoretical motivation, developing the quantum theory of ectromagnetic radiation in the absorbing medium, i. e. cosmo- gical vacuum.

Introduction

The discovery of quasars, their spectroscopic and photo- tric study have raised several new problems the solution of ich is of interest to extragalactic astronomy and cosmology well as theoretical physics. Of special interest are the values the redshift derived from the measurements of the spectral es of quasars, as they have brought out a number of rprising effects. It appeared, for example, that the quantity of the absorption-line redshift is often multiple, that the ues of z accumulate around the quantity $z=2$ and that quasars with $z>2$ are rather rare (G. R. Burbidge & M. Burbidge [1]). It is also interesting that the correlation between apparent radio brightness and the value z is almost entirely missing.

The author of the present paper has tried to explain several peculiarities of the redshifts of quasars and very remote cosmological objects in general with the quantum theory of electromagnetic waves adjusted to cosmological problems. The main features of this theory and its use in the explanation of the peculiarities of the redshifts of cosmological objects are presented in our paper "On Peculiarities of Quasar Red-Shifts and Hypotheses of Explain Them" [2]. The aim of the present paper is to give the theory of cosmological quantum electrodynamics a deeper theoretical motivation than has been done in the above-mentioned investigation. For a more general survey a brief summary of the paper mentioned will be given below.

The redshift of the spectral lines of cosmological objects is almost without exception interpreted as a result of the Doppler effect. The idea of the redshift being a result of the light quantum "fatigue", the continuous loss of energy in the course of long-time travelling in traversing cosmological distances has been expressed very seldom. Contemporary theoretical physics does not seem to offer any clue to the explanation of this phenomenon. That is why the hypothesis of the continuous loss of light quantum energy as a cause of the redshift has not been paid attention to.

On the other hand, the most characteristic feature of contemporary physics lies in assigning different physical properties to the vacuum. For instance, the electric field polarizes the vacuum analogically to a dielectric and there is nothing peculiar if the vacuum is also a conductor of electricity analogically to a medium conducting electricity.

The supposition concerning the electric conductivity of the vacuum arouses attention also because it follows, at least formally, from the covariant form of the Maxwell equations written for an isotropic and homogeneous universe, e. g. for the de Sitter universe. It appears that these equations have a certain additional cosmological term which can be interpreted in short as the conduction current in the medium whose electric conductivity coefficient σ is constant. This extremely small quantity serves also as the electric conductivity coefficient of the cosmological vacuum. However, the medium conducting electricity absorbs the electromagnetic wave energy passing through the medium. Consequently, the cosmological vacuum possesses the ability to absorb the radiation propagating in it.

Since the electric conductivity coefficient of the vacuum σ is constant, the Maxwell equations together with the additional cosmological term are still linear. They can be successfully solved by means of the Fourier method by which the waves should be treated as the superposition of monchromatic waves. In the case of an ordinary (not a cosmological) vacuum it follows from the Maxwell equations that the previously mentioned monochromatic waves have a constant amplitude and a time-independent frequency. Consideration of the cosmological term in the equations, however, shows that the amplitude of electromagnetic waves decreases exponentially with time t. This is where the ability of the cosmological vacuum to absorb the radiation passing through it becomes apparent. On the other hand, the frequency of the waves is still a time-independent constant. Since the redshift effect of the spectral lines of distant cosmological objects finds its expression in the change of the frequency of waves, the Maxwell equations cannot give, as expected, the redshift formula even with the cosmological term.

As is generally known, the formula of the redshift is not derived by means of the Maxwell equations. The redshift formula can be found by using the basic metric form. By means of that the relation of the light wavelength as well as frequency at the points of the universe far from one another are calculated. As a result we get the redshift formula. The fact that the difference of the metric tensor $g_{\mu\nu}$ from its pseudo-Euclidean value gives the additional cosmological term in the Maxwell equations, although despite this additional term the redshift effect cannot be explained by these equations, has evidently a deeper reason. The theory of electromagnetic waves in its classical form is apparently not suitable for solving some cosmological problems.

The Maxwell equations emphasize the wave aspect of electromagnetic radiation. The corpuscular character of the radiation becomes evident only through quantizing these equations. A quantized electromagnetic wave consists of photons, the photon energy being the product of its frequency and the Planck constant. Photon energy can decrease only with a decrease in frequency. Therefore, quantizing the Maxwell equations with the additional cosmological term it is to be expected that they should also give the redshift formula.

A closer examination of the problem indicates the fact that according to the quantized Maxwell equations the energy of the electromagnetic wave in the cosmological vacuum may decrease in two possible ways. It may take place with a decrease in frequency, as mentioned already, as well as with a decrease in the photon number related to the wave. In the former case the process is continuous in time, in the latter discrete. In the work under review [2] the possibility of the two processes was shown and the assumption was checked by observational data. Almost all the peculiarities of the redshift of quasars were explained there.

In the same investigation the Maxwell equations have been quantized by the method in use in the quantum theory. The radiation field is treated as an ensemble of harmonic oscillators which in a certain quantum state have the quantum number n and the energy

$$E=\hbar\omega\left(n+\frac{1}{2}\right), \tag{0.1}$$

where ω is the frequency of the oscillator as well as that of the corresponding monochromatic wave. In the classical vacuum, not in the cosmological one, both n and ω are time-independent constants.

The ability of the vacuum to be a conductor and hence an absorber of energy is expressed by the fact that both n and

ω change in time. For the frequency ω the following formula holds true:

$$\omega = \omega_0 \exp[-4\pi\sigma(t - t_0)], \tag{0.2}$$

where ω_0 is the frequency of the wave considered at its formation or excitation moment $t = t_0$ and σ is the vacuum electric conductivity coefficient. The decrease of the frequency ω with time t according to (0.2) is the redshift effect.

Formula (0.2) is valid up to the moment

$$t_1 = t_0 + \Delta t_n; \qquad \Delta t_n = \frac{1}{4\pi\sigma} \ln \frac{2n+1}{2n-1} \tag{0.3}$$

by which according to (0.2) and (0.3) the wave energy will have decreased due to the absorption effect of the vacuum exactly by one quantum

$$\hbar\omega_0, \tag{0.4}$$

where ω_0 is the wave frequency at the excitation moment t_0. Further the process may continue in two ways.

In the first case, at the moment $t = t_1$ a change in the quantum number of the wave takes place,

$$n \to n - 1, \tag{0.5}$$

while the previous frequency ω_0 is restored. The second case gives no change in n and the decrease of the wave frequency continues according to (0.2), while $t > t_1$ then. Which of the possibilities is realized is regulated by a certain statistical law, which is typical of quantum physics.

Formulae (0.2) and (0.3) are the main result of our investigation [2] and they enable one to explain several peculiarities of the redshifts of cosmological objects. In the following paragraphs an attempt will be made to motivate these basic formulae more stringently, considerably improving the quantum theory of the electromagnetic radiation in an electricity-conducting medium. Besides, the conducting medium need not be the cosmological vacuum, it may also be some real matter. However, the theory can still be mainly applied in the field of cosmology.

§ 1. Canonical Formalism

The equations of electromagnetic waves (the Maxwell equations) in the covariant presentation are the following:

$$\frac{\partial F^{\mu\sigma}}{\partial x_\sigma}+\left\{ {\mu \atop \alpha\sigma} \right\} F^{\alpha\sigma}+\left\{ {\sigma \atop \alpha\sigma} \right\} F^{\mu\alpha}=0,$$

$$\frac{\partial F_{\mu\nu}}{\partial x_\sigma}+\frac{\partial F_{\nu\sigma}}{\partial x_\mu}+\frac{\partial F_{\sigma\mu}}{\partial x_\nu}=0, \qquad (1.1)$$

where $F_{\mu\nu}$ is the electromagnetic field tensor, the other symbols are common with the general theory of relativity. The components of the metric tensor $g_{\mu\nu}$ necessary for the calculation of the Kristoffel symbols occurring in equations (1.1) are obtained from the basic metric form

$$ds^2=dx_4^2-\exp\left(\frac{8\pi}{3}\sigma\frac{x_4}{c}\right)\cdot(dx_1^2+dx_2^2+dx_3^2), \qquad (1.2)$$

which determines the space-time metric. Here $x_4=ct$, where t is time and c is the velocity of light. The constant σ is interpreted as the vacuum electric conductivity coefficient in the present paper. In the cosmological theories that are based on the basic metric form (1.2) the constant σ is proportional to the Hubble constant

$$H=\frac{4\pi}{3}\sigma=2.4\cdot10^{-18}\ sec^{-1}.$$

The basic metric form (1.2) provides the basis for the so-called steady-state cosmology developed mainly by Bondi and Gold (Bondi and Gold 1949) but for the first time used by W. de Sitter in his studies [4].

Using, for the sake of clarity, the electric field vector $\mathfrak{E}$ and the magnetic field vector $\mathfrak{H}$ instead of the electromagnetic field tensor $F_{\nu\mu}$, and applying the usual symbols of vector analysis, equation system (1.1) can be obtained in the following form:

$$
\begin{aligned}
&\operatorname{rot}\mathfrak{H}-\frac{1}{c}\frac{\partial}{\partial t}\mathfrak{E}-\frac{4\pi\sigma}{c}\mathfrak{E}=0,\\
&\operatorname{rot}\mathfrak{E}+\frac{1}{c}\frac{\partial}{\partial t}\mathfrak{H}=0,\\
&\operatorname{div}\mathfrak{E}=0,\\
&\operatorname{div}\mathfrak{H}=0.
\end{aligned}
\tag{1.3}
$$

Here and further Gothic letters are used to denote vectors.

To solve equation system (1.3), a well-known technique with the application of the vector potential $\mathfrak{A}$ and the scalar potential Φ is used,

$$
\begin{aligned}
&\mathfrak{E}=-\frac{1}{c}\frac{\partial\mathfrak{A}}{\partial t}-\operatorname{grad}\Phi,\\
&\mathfrak{H}=\operatorname{rot}\mathfrak{A},
\end{aligned}
\tag{1.4}
$$

while the supplementary Lorentz condition should hold true:

$$
\operatorname{div}\mathfrak{A}+\frac{1}{c}\frac{\partial\Phi}{\partial t}+\frac{4\pi\sigma}{c}\Phi=0.
\tag{1.5}
$$

Then the Maxwell equations (1.3) yield

$$
\begin{aligned}
&-\Delta\mathfrak{A}+\frac{1}{c^2}\frac{\partial^2\mathfrak{A}}{\partial t^2}+\frac{4\pi\sigma}{c^2}\frac{\partial\mathfrak{A}}{\partial t}=0,\\
&-\Delta\Phi+\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}+\frac{4\pi\sigma}{c^2}\frac{\partial\Phi}{\partial t}=0.
\end{aligned}
\tag{1.6}
$$

Applying the Fourier method for solving (1.6), we write

$$
\mathfrak{A}=\frac{c}{(2\pi)^3}\int\frac{1}{\omega_k^{1/2}}\mathfrak{q}_k\cdot\exp[i(kx)]\cdot\exp[2\pi\sigma(t-t_0)]\cdot d^3k
\tag{1.7}
$$

while

$$
\begin{aligned}
&\omega_k=c\,|k|=c\,(k_1^2+k_2^2+k_3^2)^{1/2},\\
&d^3k=dk_1\,dk_2\,dk_3.
\end{aligned}
\tag{1.8}
$$

The Fourier component, vector $\mathfrak{q}_k$, is actually supplied with three indices $k_1k_2k_3$. But for the sake of brevity

$$
\mathfrak{q}_{k_1,k_2,k_3}=\mathfrak{q}_k,
$$

the abbreviation being used also further on.

The symbol

$$(kx)=x_1k_1+x_2k_2+x_3k_3$$

is the scalar production of the vectors $x\equiv(x_1x_2x_3)$ and $k\equiv(k_1k_2k_3)$. (By way of exception Latin letters are used to denote vectors).

On the analogy of formula (1.7) for the scalar potential Φ is also written

$$\Phi=\frac{c}{(2\pi)^3}\int\frac{1}{\omega_k^{1/2}}\varphi_k\cdot\exp[i(kx)]\cdot\exp[-2\pi\sigma(t-t_0)]\cdot d^3k. \quad (1.9)$$

The Fourier components, the vector $\mathfrak{q}_k$ and the scalar φ_k are the functions of time t. The following equations hold:

$$\begin{aligned}\ddot{\mathfrak{q}}_k+\varkappa_k^2\,\mathfrak{q}_k&=0,\\ \ddot{\varphi}_k+\varkappa_k^2\,\varphi_k&=0,\end{aligned} \quad (1.10)$$

while

$$\varkappa^2=c^2|k|^2-4\pi^2\sigma^2=\omega^2-4\pi^2\sigma^2. \quad (1.11)$$

From the additional conditions (1.5) the following relation is added to equations (1.10):

$$i(k\mathfrak{q}_k)+\frac{2\pi\sigma}{c}\varphi_k+\frac{1}{c}\dot{\varphi}_k=0. \quad (1.12)$$

To satisfy equation (1.12), three unit vectors are used

$$\mathfrak{e}_k^{(1)},\quad \mathfrak{e}_k^{(2)},\quad \mathfrak{e}_k^{(3)}, \quad (1.13)$$

which are defined as follows: the vector $\mathfrak{e}_k^{(1)}$ is chosen in the direction of the vector $k\equiv(k_1k_2k_3)$, so that

$$k=|k|\cdot\mathfrak{e}_k^{(1)}. \quad (1.14)$$

The other two vectors $\mathfrak{e}^{(2)}$ and $\mathfrak{e}^{(3)}$ while being orthogonal to each other are also orthogonal to the vector $\mathfrak{e}_k{}^{(i)}$.

Fourier integrals (1.7) and (1.9) indicate that electromagnetic waves are regarded as a result of the superposition of single waves with a definite wave-length, frequency and propagation direction. The vector $e_k{}^{(1)}$ is in the direction of propagation of the wave, the vectors $e_k^{(2)}$ and $e_k^{(3)}$ are orthogonal to it. They determine the wave polarization plane and are the wave polarization vectors.

It is obvious that each vector $\mathfrak{q}_k$ can be expressed as follows:

$$\mathfrak{q}_k = q_k^{(1)} \cdot \mathfrak{e}_k^{(1)} + q_k^{(2)} \cdot \mathfrak{e}_k^{(2)} + q_k^{(3)} \cdot \mathfrak{e}_k^{(3)}, \tag{1.15}$$

where the quantities $q_k^{(s)}$ are the projections of the vector $\mathfrak{q}_k$ to the axes determined by the unit vectors $\mathfrak{e}_k^{(s)}$. From equations (1.10) it is easy to obtain

$$\ddot{q}_k^{(s)} + \varkappa_k^2 q_k^{(s)} = 0, \qquad s = 1; 2; 3 \tag{1.16}$$

$$\varkappa_k^2 = \omega_k^2 - 4\pi^2\sigma^2,$$

while the supplementary condition (1.12) yields:

$$i\omega_k q_k^{(1)} + 4\pi\sigma\varphi_k + \dot{\varphi}_k = 0. \tag{1.17}$$

The supplementary condition relates the Fourier components $q_k^{(1)}$ and φ^k, it does not concern the components $q_k^{(2)}$ and $q_k^{(3)}$ Therefore, to study the electromagnetic radiation field it is sufficient to deal with the components $q_k^{(2)}$ and $q_k^{(3)}$ and consider the supplementary condition fulfilled automatically.

If we take

$$H_k^{(s)} = \frac{1}{2}\{\omega_k (p_k^{(s)2} + q_k^{(s)2}) + 2\pi\sigma (p_k^{(s)} q_k^{(s)} + q_k^{(s)} p_k^{(s)})\} \tag{1.18}$$

as the Hamilton function, equations (1.16), where $s = 2; 3$, can be presented in canonical form. Here the Hamiltonian of the whole field is the sum of the Hamiltonians of the single waves

$$H = \sum_{s=2}^{3} \int H_k^{(s)} d^3k. \tag{1.19}$$

The quantites $p_k^{(s)}$ and $q_k^{(s)}$ are the canonical variables of the field. The Hamilton canonical equations

$$\frac{\partial H}{\partial p_k^{(s)}} = \dot{q}_k^{(s)}, \qquad \frac{\partial H}{\partial q_k^{(s)}} = -\dot{p}_k^{(s)} \tag{1.20}$$

give equations (1.16). In canonical variables the vectors $\mathfrak{E}$ and $\mathfrak{H}$ of the electric and magnetic fields can be expressed as

$$\mathfrak{E}=-\sum_{s=2}^{3}\frac{1}{(2\pi)^3}\int\mathfrak{E}_k^{(s)}\exp[i(kx)]\cdot\exp[-2\pi\sigma(t-t_0)]\cdot d^3k, \quad (1.21)$$

$$\mathfrak{H}=\sum_{s=2}^{3}\frac{1}{(2\pi)^3}\int\mathfrak{H}_k^{(s)}\cdot\exp[i(kx)]\cdot\exp[-2\pi\sigma(t-t_0)]\cdot d^3k, \quad (1.22)$$

while

$$\mathfrak{E}_k^{(s)}=-\mathfrak{e}_k^{(s)}\cdot\omega_k^{1/2}p_k^{(s)}, \quad (1.23)$$

$$\mathfrak{H}_k^{(s)}=\frac{1}{i}[\mathfrak{e}_k^{(s)}\times\mathfrak{e}_k^{(1)}]\cdot\omega_k^{1/2}q_k^{(s)}, \quad (1.24)$$

and the symbol

$$[\mathfrak{e}_k^{(s)}\times\mathfrak{e}_k^{(1)}]$$

is the vectorial product of the vectors $\mathfrak{e}_k^{(s)}$ and $\mathfrak{e}_k^{(1)}$. The energy of the electromagnetic field

$$E=\frac{1}{8\pi}\int\{\mathfrak{E}^2+\mathfrak{H}^2\}d^3k \quad (1.26)$$

can be calculated by means of formulae (1.21) and (1.22). As a result, we obtain

$$E=\frac{1}{(2\pi)^3}\sum_{s=2}^{3}\int E_k^{(s)}d^3k, \quad (1.27)$$

where

$$E_k^{(s)}=\frac{1}{2}\omega_k(p_k^{(s)2}+q_k^{(s)2})\cdot\exp[-4\pi\sigma(t-t_0)]. \quad (1.28)$$

The energy $E_k^{(s)}$ given by formula (1.28) is that of a single wave. According to (1.27), the energy of the whole field is the sum of the energies of all the single waves.

Contrary to the Hamiltonian $H_k^{(s)}$ the energy $E_k^{(s)}$ depends explicitly on the time t, namely through the exponential function in (1.28).

In what follows we shall mainly deal with single waves and their energies. For the sake of simplicity it is expedient to drop the indices denoting a single wave, e. g. instead of $q_k^{(s)}$ one may write q, etc. Such a simplification should not cause any confusion. If there is still a danger of misunderstanding, the indices are restored in their previous form.

After calculating the partial derivatives the Hamiltonian canonical equations are the following:

$$\dot{q}=\omega p+2\pi\sigma q, \qquad -\dot{p}=\omega q+2\pi\sigma p \tag{1.29}$$

the solutions of which are

$$q=\left(\frac{\hbar\omega}{2\varkappa}\right)^{1/2}\cdot\{a\cdot\exp[i\varkappa t]+a^{+}\cdot\exp[-i\varkappa t]\},$$
$$-p=\left(\frac{\hbar}{2\omega\varkappa}\right)^{1/2}\cdot\{(2\pi\sigma-i\varkappa)a\cdot\exp[i\varkappa t]+(2\pi\sigma+i\varkappa)a^{+}\cdot\exp[-i\varkappa t]\}, \tag{1.30}$$

where a and a^{+} are integration constants. Using the solutions (1.30), the Hamiltonian and the energy can be calculated. The results of the calculations are

$$H=\hbar\omega\cdot\frac{1}{2}(aa^{+}+a^{+}a), \tag{1.31}$$

$$E=\frac{\hbar}{2\varkappa}\cdot\{\omega^{2}(aa^{+}+a^{+}a)+2\pi\sigma(2\pi\sigma-i\varkappa)a^{2}\cdot\exp[2i\varkappa t]+$$
$$+2\pi\sigma(2\pi\sigma+i\varkappa)a^{+2}\cdot\exp[-2i\varkappa t]\}\cdot\exp[-4\pi\sigma(t-t_0)]. \tag{1.32}$$

It should be remembered that indices have been dropped in these formulae. The Hamiltonian (1.31) and the energy (1.32) are those of a single wave.

It deserves emphasizing that according to (1.31) H is time-independent or, in other words, the constant of the problem. The energy, however, depends on the time t due to the occurrence of the periodic terms

$$\exp(2i\varkappa t), \quad \exp(-2i\varkappa t) \tag{1.33}$$

as well as the term exponentially decreasing in time

$$\exp[-4\pi\sigma(t-t_0)] \tag{1.34}$$

in the corresponding formula. The dependence through the periodic terms is after all inssential. Indeed, to measure the wave energy one must spend time considerably longer than is the duration of one oscillation period,

$$P=\frac{2\pi}{\varkappa}.$$

The mean value of energy for this time interval is

$$\bar{E}=\frac{\hbar}{2\varkappa}\omega^2(aa^+ + a^+a)\cdot\exp[-4\pi\sigma(t-t_0)], \qquad (1.35)$$

since the mean values of periodic terms (1.33) are zeros.

§ 2. Quantum Theory of Electromagnetic Waves in a Conducting Medium. First Approximation

With the presentation of the equations of electromagnetic waves in canonical form the necessary preliminary work has been done to quantize the electromagnetic waves in a medium conducting electricity. Following the traditional quantum theory of fields, the field canonical variables p and q must be treated as the operators satisfying the basic commutation rules:

$$[p_k^{(s)}, p_{k'}^{(s')}]=[q_k^{(s)}, q_{k'}^{(s')}]=0,$$
$$[p_k^{(s)}, q_k^{(s')}]=\frac{\hbar}{i}\delta^3(k-k')\cdot\delta_{ss'} \qquad (2.1)$$

while

$$\delta^3(k-k')=\delta(k_1-k'_1)\delta(k_2-k'_2)\delta(k_3-k'_3),$$
$$[p,q]=pq-qp.$$

Being the solutions of the Hamilton canonical equations, the operators p and q depend on time as given in expressions (1.30). Here a and a^+ are integration constants which should now be regarded as time-independent operators. Relying on formulae (2.1), it is possible to derive the commutation rule

$$[a, a^+]=1. \qquad (2.2)$$

In the traditional quantum theory of the electromagnetic field the operators a and a^+ are those of creation and annihilation of photons. On the analogy of the traditional quantum theory of fields the operator

$$\mathfrak{N}=aa^+ \tag{2.3}$$

is also called the particle number operator, here, however, the photon number operator. Its eigenvalues are all positive integers, including zero. The Hamiltonian is related to the operator by a simple formula

$$H=\hbar\varkappa\left(\mathfrak{N}+\frac{1}{2}\right). \tag{2.4}$$

The energy operator E may also be expressed by using the operator $\mathfrak{N}$ in the following way:

$$E=\frac{\hbar}{2\varkappa}\{\omega^2(2\mathfrak{N}+1)+2\pi\sigma(2\pi\sigma-i\varkappa)a^2\cdot\exp[2i\varkappa t]+$$
$$+2\pi\sigma(2\pi\sigma+i\varkappa)a^{+2}\cdot\exp[-2i\varkappa t]\}\cdot\exp[-4\pi\sigma(t-t_0)]. \tag{2.5}$$

In the derivation of this formula (1.32) and (2.3) have been used.

The Hamiltonian (2.4) is independent of the time t. In contrast to the Hamiltonian the energy operator depends on time and this dependence is expressed in (2.5) by the periodic terms as well as by the factor exponentially decreasing with time. However, taking the time average of the energy operator E, as in deriving formula (1.34), we get

$$\overline{E}=\frac{1}{2}\hbar\omega\left(\mathfrak{N}+\frac{1}{2}\right)\cdot\exp[-4\pi\sigma(t-t_0)], \tag{2.6}$$

in which we substituted

$$\varkappa=\omega.$$

Such a substitution simplifies the corresponding formulae and it is justified in the case of cosmological problems since the error caused is negligible. So, below (2.6) will be used as an expression of the energy operator because of its simplicity and easier interpretation. It should be noted that in the presentation where $\mathfrak{N}$ is diagonal, the operator $\overline{E}$ is also diagonal.

Let us interpret the results obtained.

An electromagnetic wave with the frequency ω is excited at the moment $t=t_0$. Then a wave consisting of n photons appears and the energy of the wave is

$$\bar{E}_0=\hbar\omega\left(n+\frac{1}{2}\right), \tag{2.7}$$

since the exponential factor in (2.6) is unity and n is the corresponding eigenvalue of the operator $\mathfrak{N}$. At the following moments $t>t_0$ the photon number n is constant since the operator $\mathfrak{N}$ and its eigenvalue do not depend on time. The energy, however, decreases according to the following formula:

$$\bar{E}=\bar{E}_0\cdot\exp[-4\pi\sigma(t-t_0)]=\hbar\omega\cdot\left(n+\frac{1}{2}\right)\cdot\exp[-4\pi\sigma(t-t_0)]. \tag{2.8}$$

The energy of a single photon is

$$e=\hbar\omega\cdot\exp[-4\pi\sigma(t-t_0)] \tag{2.9}$$

and it also decreases exponentially with the time t.

The theory developed up to here in the present paper is an application of the traditional quantum theory of fields to the investigation of the electromagnetic radiation in conducting medium. Up to now not a single new idea has been used which has not been applied in the already well-established quantum physics. The results have been summed up in (2.8). However, this formula is not in accordance with the observational data and, above all, with the redshift law of the spectral lines of cosmological objects.

One of the basic statements in quantum physics is that the energy of a photon is the product of the Planck constant and the frequency of the photon,

$$e=\hbar\omega. \tag{2.10}$$

According to this, the energy of a photon decreases only at the expense of a decrease in frequency, not as given in (2.9), where the frequency ω is an invariable quantity. In the author's previous study [2] this contradiction was removed by an independent postulate according to which at the moment t the actually observed frequency $\bar{\omega}$ is related to the frequency ω through the formula

$$\bar{\omega}=\omega\cdot\exp[-4\pi\sigma(t-t_0)]. \tag{2.11}$$

Consequently, in (2.9) the observable quantity is not ω but the product of ω and $e^{-\pi\sigma(t-t_0)}$ according to (2.11). This postulate, intuitively introduced into the theory, requires more motivation which we intend to give in the following chapter.

§ 3. Quantum Theory of Electromagnetic Waves in an Electricity Conducting Medium. Second Approximation

The quantization of the Maxwell equations did not yield the desired results at first. The redshift formula (2.11) could be introduced into the theory only as an independent postulate. In order to give some deeper reasons for this postulate, it is necessary to analyze these differences in the problem caused by a conducting medium as compared with the problem where the classical vacuum serves as a medium.

In the quantum theory of the radiation field an important role is played by the Hamiltonian and the energy operator. In a non-conducting medium they both are equal quantities not depending on the time t.

The situation changes when the field is in an electricity-conducting and consequently energy-absorbing medium. The Hamiltonian and the energy are different whereas the energy expression contains the time t in an explicit form. This circumstance is of decisive importance for the further development of the quantum theory of the radiation field.

Let us analyze the problem providing that in the beginning electromagnetic radiation is in an ordinary vacuum which does not absorb this radiation. In this case the Hamiltonian and the energy are identical.

The radiation field is regarded as an ensemble of harmon oscillators. An oscillator corresponds to each monochromat wave and the investigation of the radiation field is reduce to the study of oscillators. All the basic qualities and motic rules of the oscillator, hence also those of a monochromat wave, are obtained from the solutions of the Schröding equation

$$H(p, q)\Psi = -\frac{\hbar}{i}\frac{\partial \Psi}{\partial t}, \qquad (3.1$$

which is the basic equation of the wave theory of matter. Th function $\Psi(q, t)$ is the wave function and $H(p, q)$ in th equation is the Hamiltonian, which in its turn is a functio of canonical variables. p is usually the differentiating operatc

$$p = \frac{\hbar}{i}\frac{\partial}{\partial q}.$$

In such a case q is an ordinary c-number.

By means of its eigenvalues wave equation (3.1) gives th quantized energy of the wave as well as the Hamiltonia identical to it. To each state of quantized energy correspond a solution of the Schrödinger equation

$$\Psi = \psi_n \cdot \exp[i\omega_n t], \qquad (3.2$$

where $\psi_n(q)$ is the n-*th* eigenfunction and ω_n is the frequenc of the corresponding waves of matter. It is related to the valu of quantized energy through the formula

$$E_n = \hbar\omega_n. \qquad (3.3$$

As we can see, the frequency of the waves of matter i determined through the eigenvalues of the Hamiltonian as we as through the eigenvalues of the energy in case th Hamiltonian and the energy are identical.

In the formalism of the quantum theory the co-ordinate an moment operators p and q form the so-called canonical pair they are canonically dual to one another. The latter means also that between these operators the basic relation of the quantum theory is valid in the form of the corresponding commutation rule

$$[p, q] = \frac{\hbar}{i},$$

which permits, for instance, to regard the momentum operator as a differentiating one

$$p = \frac{\hbar}{i} \frac{\partial}{\partial q}$$

and q as an ordinary c-number.

For the Hamiltonian operator H time t is canonically dual. If we regard time t as a c-number, then

$$H = -\frac{\hbar}{i} \frac{\partial}{\partial t}. \tag{3.4}$$

On the other hand, in the problem of the motion of a particle the Hamiltonian can be given as a function of the operators p and q. Taking this fact into consideration and applying (3.4) to the function Ψ, the Schrödinger equation (3.1) will be obtained.

Since the frequency of the waves of matter is given by the corresponding solution of the Schrödinger equation, it is also determined by the Hamiltonian of the problem.

Complications arise when the Hamiltonian and the energy are not identical. In this case too the Schrödinger equation formed by means of the Hamiltonian gives the waves and their frequency $\varkappa$. But when measuring electromagnetic waves, their energy and not the Hamiltonian is measured. However, in measuring the energy one does not obtain the frequency of the waves $\varkappa$, which is determined by the Hamiltonian. It is to be supposed that there exists another wave equation related to the energy and that the frequency of the waves determined by this wave equation is the actual result of the measurements. This, however, means also that there exists a canonically dual

τ for the operator E, just as the Hamiltonian has a canonically dual t. In this case

$$E = \frac{\hbar}{i} \frac{\partial}{\partial \tau} \tag{3.5}$$

and the wave equation formed by means of the energy operator E would be

$$E \cdot \Phi = -\frac{\hbar}{i} \frac{\partial \Phi}{\partial \tau} \tag{3.6}$$

where τ is a certain c-number, a quantity measuring the time current.

The operator E contains the time t explicitly. Simultaneously with the wave equation (3.6) another time-measuring quantity τ has been introduced into the theory. Questions arise as to the relations between the quantities τ and t and how one should understand the existence of the two time measuring quantities in the theory.

First of all, it should be noted that as a result of the discussion presented above two pairs of canonically dual operators

$$H, t; \quad E, \tau$$

are in the focus of attention. These operators do not commute with each other and cannot therefore be given simultaneously in diagonal form. While the time τ is regarded in the theory as a c-number, H, t and E are not c-numbers any longer. Thus, in wave equation (3.6), where t appears in the operator E explicitly, t is not an ordinary c-number either but a quantity not commutable with several other operators. When the idea is developed further, a number of conceptual complications disappear which at first sight seem to arise due to the existance of two time current measuring quantities in the theory.

§ 4. Time Operator $\hat{t}$ and its Basic Properties

Let us define the operator $\hat{t}$ as canonically dual to the Hamiltonian H. Let the latter not contain t explicitly. To find a dual $\hat{t}$ as a function of p and q for a concrete H, we shall first consider the problem from the point of view of macrophysics, in order to re-interpret the relations afterwards in terms of microphysics.

Let $A(p, q)$ and $B(p, q)$ be two functions of the canonical variables p and q. They are canonically dual with respect to one another if the Poisson brackets formed from them are unity:

$$[A,B]\equiv\frac{\partial A}{\partial p}\cdot\frac{\partial B}{\partial q}-\frac{\partial A}{\partial q}\cdot\frac{\partial B}{\partial p}=1. \qquad (4.1)$$

The quantum-theoretical equivalent of the Poisson brackets is the A and B commutator. In the quantum theory formula (4.1) is written

$$[A, B]=AB-BA=\frac{\hbar}{i}.$$

If H and t are canonically dual, then according to (4.1)

$$\frac{\partial H}{\partial p}\cdot\frac{\partial t}{\partial q}-\frac{\partial H}{\partial q}\cdot\frac{\partial t}{\partial p}=1 \qquad (4.2)$$

should hold. Since

$$H=\frac{1}{2}\{\omega(p^2+q^2)+2\pi\sigma(pq+qp)\}, \qquad (4.3)$$

(4.2) gives the differential equation for the quantity t:

$$(\omega p+2\pi\sigma q)\frac{\partial t}{\partial q}-(\omega q+2\pi\sigma p)\frac{\partial t}{\partial p}=1. \qquad (4.4)$$

The most suitable form for this problem is the following solution of the equation:

$$t=\frac{1}{i\varkappa}\ln\left\{\frac{(2\pi\sigma+i\varkappa)q+\omega p}{\omega^{1/2}(\omega(p^2+q^2)+2\pi\sigma(pq+qp))^{1/2}}\right\}+t_0, \qquad (4.5)$$

where t_0 is the integration constant and

$$\varkappa^2=\omega^2-4\pi^2\sigma^2.$$

Time t given by (4.5) depends on the canonical variables p and q. On the other hand, when regarded as the solutions of the Hamilton canonical equations, p and q are functions of the variable t. Therefore, substituting into (4.5) instead of p and q their expressions as the solutions of canonical equations we should obtain identity. The following will convince us of this.

The Hamilton canonical equations are

$$\frac{\partial H}{\partial p}=\dot{q} \quad \text{or} \quad \frac{dq}{dt}=\omega p+2\pi\sigma q,$$
$$\frac{\partial H}{\partial p}=-\dot{p} \quad \text{or} \quad -\frac{dp}{dt}=\omega q+2\pi\sigma p, \tag{4.6}$$

where H has been taken from formula (4.3). It follows that

$$\frac{(2\pi\sigma+i\varkappa)q+\omega p}{\omega^{1/2}(\omega(p^2+q^2)+2\pi\sigma(pq+qp))^{1/2}}=i\frac{M}{(MM^*)^{1/2}}\cdot\exp[i\varkappa t]$$
$$\frac{(2\pi\sigma-i\varkappa)q+\omega p}{\omega^{1/2}(\omega(p^2+q^2)+2\pi\sigma(pq+qp))^{1/2}}=-i\frac{M^*}{(MM^*)^{1/2}}\cdot\exp[-i\varkappa t], \tag{4.7}$$

where M and M^* are integration constants. When using (4.7), it is easy to note that by a proper choice of the constants equations (4.5) become identical. At the same time it is shown that according to (4.5) $t(pq)$ is a time-measuring quantity.

With a view to obtaining the quantum-theoretical operator $\hat{t}$ from (4.5) one should regard p and q in this expression as the corresponding quantum-theoretical variables subjected to the commutation rules (2.1). In this case, however, one should define more exactly how to understand the symbols of the functions occurring in (4.5), above all what the logarithm of a rather complicated expression of the operators p and q means. For this purpose let us consider t given by (4.5) once more as a quantity of macrophysics and have a try at finding the power series that would determine the function $t(pq)$. As we know, it is possible to re-interpret a power series in an operator series. Let us denote

$$\frac{\varkappa q}{\omega^{1/2}(\omega(p^2+q^2)+2\pi\sigma(pq+qp))^{1/2}}=X, \qquad (4.8)$$

then

$$\frac{2\pi\sigma q+\omega p}{\omega^{1/2}(\omega(p^2+q^2)+2\pi\sigma(pq+qp))^{1/2}}=(1-X^2)^{1/2} \qquad (4.9)$$

and formula (4.5) may be represented as follows:

$$t-t_0=-\frac{i}{\varkappa}\ln\{iX+(1-X^2)^{1/2}\}=\frac{2m\pi}{\varkappa}+\frac{1}{\varkappa}\arcsin X. \qquad (4.10)$$

Here m is a positive or negative integer and the function arcsin X is defined as a power series

$$\arcsin X=X+\frac{1}{2\cdot 3}X^3+\frac{1\cdot 3}{2\cdot 4\cdot 5}X^5+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7}X^7+\ldots \qquad (4.11)$$

or

$$\arcsin X=\pi-\left(X+\frac{1}{2\cdot 3}X^3+\frac{1\cdot 3}{2\cdot 4\cdot 5}X^5+\ldots\right). \qquad (4.12)$$

Series (4.11) and (4.12) converge in the interval $-1\leqslant X\leqslant 1$.

With $X=\pm 1$ both series give an identical value for the function arcsin X. For other values arcsin X is double-valued.

The presentation of (4.5) through formulae (4.8) and series (4.11) and (4.12) makes it possible to define the operator $\hat{t}$ as a certain function of the operators p and q. Therefore, let us note first that according to (4.8) the quantity X can also be written as follows:

$$X=\frac{\varkappa q}{\omega^{1/2}H^{1/2}}, \qquad (4.13)$$

while the expression of the Hamiltonian according to (4.3) has been kept in mind. It is especially simple to calculate the matrix X in the presentation where the Hamiltonian H is diagonal. In this case the matrix $H^{-1/2}$ is also diagonal and diagonal terms can be calculated easily. In view of this circumstance, the calculation of the matrix X by means of formula (4.13) is an easy task. If there is an expression for X, it is also for X^n where n is an integer. Now expansions in series (4.11) and (4.12) define also $\arcsin X$ as a matrix function of the matrix X.

According to the presented discussion the operator $\hat{t}$ is the following:

$$\hat{t} - t_0 = \frac{2\pi}{\varkappa} \cdot m + \frac{1}{\varkappa} \arcsin X. \tag{4.14}$$

As it appears from this, the time operator $\hat{t}$ consists of two addends, the former being an ordinary c-number, the latter an operator with the eigenvalues in the range of $0 \ldots 2\pi$. This result has a clear quantum-theoretical interpretation, which lies in the following.

The current of the time t is reflected in the operator (4.14) by the sequence of the integers m. If the sequence begins with the number $m=0$, the time current can be reflected by the numbers 0, 1, 2, 3 ... This part of the operator $\hat{t}$ which can be given as a c-number, gives a discrete time current, the size of a jump

$$P = \frac{2\pi}{\varkappa}$$

being equal to a period of the quantum-theoretical system considered. Consequently, the c-number part of the time operator measures the time current by periods. The other part of the operator $\hat{t}$ in (4.14) is the matrix. It reflects the time current during a wave period. From the point of view of the quantum theory, it is clear that in a strict microphysical sense it is impossible to give a state of a particle of matter, (e. g. the state of a photon, its energy, etc.) during a shorter time-interval than the corresponding wave period. This is in what the structure of (4.14) is reflected, i. e. the time current measured by the duration of the periods is a c-number, the time current with a shorter duration than the period P is a matrix.

§ 5. Second Schrödinger Equation of Electromagnetic Waves

The electromagnetic wave energy is

$$E=\frac{1}{2}\omega(p^2+q^2)\cdot\exp[-4\pi\sigma(t-t_0)]. \tag{5.1}$$

This formula follows from the Maxwell equations with an additional cosmological term. Besides the Hamiltonian, energy expression (5.1) is one of the most essential quantities characterizing the radiation field. In the case of a radiation-absorbing medium one should, besides the wave equation formed by means of the Hamiltonian, also take into account the wave equation obtained through the energy operator. Let us call the wave equation obtained by means of the Hamiltonian the first, and that obtained by means of the energy operator the second Schrödinger equation.

The necessity of the second Schrödinger equation in the theory was already motivated above and the corresponding equation was given by (3.6). For its concrete presentation the energy expression (5.1) should be used where p, q and $\hat{t}$ are the corresponding operators. Relying on formula (4.14) we can write

$$\exp[-4\pi\sigma(\hat{t}-t_0)]=\exp\left[-4\pi\sigma\frac{2\pi}{\varkappa}m\right]\cdot\exp\left[-\frac{4\pi\sigma}{\varkappa}\arcsin X\right]=$$

$$=\exp\left[-\frac{8\pi^2\sigma}{\varkappa}m\right]\cdot\left(1-4\pi\sigma\frac{q}{\omega^{1/2}H^{1/2}}+\dots\right), \tag{5.2}$$

in which also formula (4.13) has been used. The operator can now be expressed as follows:

$$E=\frac{1}{2}\omega(p^2+q^2)\cdot\exp\left[-\frac{8\pi^2\sigma}{\varkappa}m\right]\cdot\left(1-4\pi\sigma\frac{q}{\omega^{1/2}H^{1/2}}\right), \tag{5.3}$$

whereas the terms of higher order of the series expansion have been left out of consideration.

Let τ, which is canonically dual to the energy, be a c-number. The second Schrödinger equation reads

$$E(p,q)\cdot\Phi(q,\tau)=-\frac{\hbar}{i}\frac{\partial}{\partial\tau}\Phi(q,\tau),$$

or keeping in mind formula (5.3), it may read

$$\frac{1}{2}\omega\left(-\hbar^2\frac{\partial^2}{\partial q^2}+q^2\right)\left(1-4\pi\sigma\frac{q}{\omega^{1/2}H^{1/2}}\right)\cdot\exp\left[-\frac{8\pi^2\sigma}{\varkappa}m\right]\cdot\Phi= -\frac{\hbar}{i}\frac{\partial\Phi}{\partial\tau}, \tag{5.4}$$

where p has been taken as follows

$$p=\frac{\hbar}{i}\frac{\partial}{\partial q}.$$

The operator E does not contain the time τ explicitly but here occurs the term

$$\exp\left[-\frac{8\pi^2\sigma}{\varkappa}m\right]$$

where the integers m measure the time current. To solve wave equation (5.4) it is necessary to clarify the inter-relations between the quantities m and τ.

Perhaps the problem will need a closer examination, but already now it is possible to propose hypotheses that should not contradict the inner logic of the theory. Namely, it is possible to suppose that m and τ as time current measuring quantities increase in parallel. However, since m is an integer, its increase proceeds discretely; τ, on the contrary, increases continuously. It means that during the rise of τ within the interval determined by the length of the period

$$P=\frac{2\pi}{\varkappa},$$

m remains constant. Thus, the quantity m is the function of the time variable τ which can be represented by a stepped graph (see Fig. 1). With this in mind it is quite easy to find solutions for (5.4) in the following way.

We substitute

$$\Phi=a_n(\tau)\cdot\varphi_n(q) \tag{5.5}$$

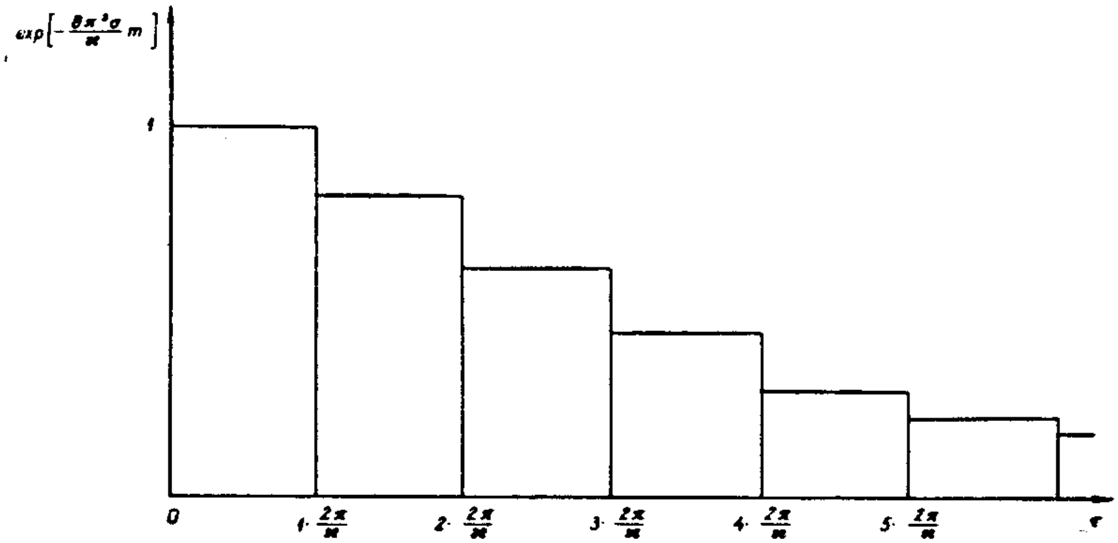

Fig. 1.

where φ_n is the solution of the equation

$$\frac{1}{2}\omega\left(-\hbar^2\frac{\partial^2}{\partial q^2}+q^2\right)\left(1-4\pi\sigma\frac{q}{\omega^{1/2}H^{1/2}}\right)\varphi_n=E_n^{(0)}\varphi_n \qquad (5.6)$$

and $E_n{}^0$ is the corresponding eigenvalue. The function of the time τ $a_n(\tau)$ in (5.5), however, should satisfy the equation

$$\frac{d}{d\tau}(\ln a_n)=-\frac{i}{\hbar}E_n^{(0)}\cdot\exp\left[-\frac{8\pi^2\sigma}{\varkappa}m\right]. \qquad (5.7)$$

On conditions (5.6) and (5.7) substitution (5.5) really serves as the solution of wave equation (5.4).

It follows from differential equation (5.7) that

$$\ln a_n=-\frac{i}{\hbar}E_n^0\int_0^\tau \exp\left[-\frac{8\pi^2\sigma}{\varkappa}m\right]d\tau. \qquad (5.8)$$

Calculating this integral, note that the quantity m as the function of the time variable τ is represented on the graph (Fig. 1). Relying upon this graph, it is possible to write

$$\ln a_n=\ln C_n(m)-\frac{i}{\hbar}E_n^0\cdot\exp\left[-\frac{8\pi^2\sigma}{\varkappa}m\right]\cdot\tau, \qquad (5.9)$$

whereas

$$\ln C_n(m) = -\frac{i}{\hbar} E^0_n \cdot \frac{2\pi}{\varkappa} \cdot \left\{ -m \cdot \exp\left[-\frac{8\pi^2\sigma}{\varkappa} m \right] + \right.$$
$$\left. + \sum_{k=1}^{m} \exp\left[-\frac{8\pi^2\sigma}{\varkappa} (k-1) \right] \right\}, \qquad (5.10)$$

when $(m-1)\frac{2\pi}{\varkappa} < \tau < m\frac{2\pi}{\varkappa}$.

The geometrical sum occurring in (5.9) and (5.10) is easliy calculable, and so we get:

$$\sum_{k=1}^{m} \exp\left[-\frac{8\pi^2\sigma}{\varkappa}(k-1) \right] = \frac{\exp\left[-\frac{8\pi^2\sigma}{\varkappa} m \right] - 1}{\exp\left[-\frac{8\pi^2\sigma}{\varkappa} \right] - 1}.$$

Having established in the form of (5.10) $a(\tau)$ as the function of the quantity τ, the solution of wave equation (5.4) according to (5.5) can be represented in the following form

$$\Phi = C_n(m) \cdot \exp\left[-\frac{i}{\hbar} E^0_n \cdot \exp\left[-\frac{8\pi^2\sigma}{\varkappa} m \right] \tau \right] \cdot \varphi_n(q). \qquad (5.11)$$

It should be mentioned that according to (5.10)

$$C^*_n(m) C_n(m) = 1.$$

The function $\varphi_n(q)$ in solution (5.11) satisfies equation (5.6) with the eigenvalue $E_n^{(0)}$.

Good results in solving equation (5.6) are obtained when using the perturbation method if we take

$$-4\pi\sigma \frac{q}{\omega^{1/2} H^{1/2}} \qquad (5.12)$$

as a perturbation term and if we use the equation

$$\frac{1}{2}\omega\left(-\hbar^2 \frac{\partial^2}{\partial q^2} + q^2 \right) \varphi^{(0)}_n = E^{(0)}_n \varphi^{(0)}_n \qquad (5.13)$$

to get the solution in the first approximation. Equation (5.13) is the Schrödinger equation of a harmonic oscillator with the eigenvalues

$$E_n^{(0)}=\hbar\omega\left(n+\frac{1}{2}\right), \qquad (5.14)$$

where $n \geqslant 0$ is an integer and eigenfunctions can be expressed in a well-known way by means of the Hermite orthogonal functions. It can also be shown that making the solutions more exact by the perturbation method the next approximation does not change the eigenvalues in (5.14) and that the correction term of the eigenfunctions is proportional to the quantity σ, consequently being a very small quantity. Taking this fact into account, one can proceed from (5.13) instead of (5.6) in most cosmological problems.

Drawing conclusions we can state on the basis of the above analysis that instead of wave equation (5.4) it is possible to use with sufficient exactness the wave equation

$$\frac{1}{2}\bar{\omega}\left(-\hbar^2\frac{\partial^2}{\partial q^2}+q^2\right)\Phi_n(q)=-\frac{\hbar}{i}\frac{\partial\Phi_n}{\partial\tau}, \qquad (5.15)$$

where

$$\bar{\omega}=\omega\cdot\exp\left[-\frac{8\pi^2\sigma}{\varkappa}m\right]=\omega\cdot\exp[-4\pi\sigma(t-t_0)]. \qquad (5.16)$$

In solving equation (5.15) the quantity m as well as the quantity related to it

$$t-t_0=\frac{2\pi}{\varkappa}m$$

may be regarded as independent of the time variable τ. Here $t-t_0$ is the time that has elapsed since the excitation of the corresponding harmonic oscillator into the observable quantum state n.

Equation (5.15) is the wave equation of the harmonic oscillator with the frequency $\bar{\omega}$. Being one of the basic formulae of the traditional quantum theory, (5.15) shows that most of the well-known formulae in the theory of electromagnetic radiation can also be transferred into the theory of electromagnetic radiation in a conducting medium after substitution (5.16) is performed. This, however, is the well-known redshift formula the quantum-theoretical motivation of which has been given by the above discussion.

§ 6. On Change in the Photon Number and Other Related Problems

The photon number operator $\mathfrak{N}$ is time-independent and the eigenvalues n of the operator are constants in time. Therefore, the energy absorption effect of the vacuum does not affect the photon number n which, once it has come into existence, remains. The question, however, is important enough to justify a closer analysis.

The process of the creation of photons might be described as follows. The electromagnetic waves with the frequency ω are excited at the moment $t = t_0$, n photons are created while the necessary amount of energy

$$n \cdot \hbar\omega \tag{6.1}$$

is obtained from a physical system present which caused the creation of the wave. After the excitation at $t > t_0$ the energy is continuously transferred from the wave into the vacuum. However, since the photon number n is constant, the amount of energy belonging to each photon decreases and, as was shown above, it takes place at the expense of the decrease in frequency.

The fact that the photon number n is constant in time may also be derived from the other considerations proceeding from the calculations of the energy balance. In the transitions

$$n \rightarrow n - 1 \tag{6.2}$$

the wave energy should also normally decrease and, namely, by the amount of the energy of the disappearing photon. In case the process takes place in the vacuum where there is no other physical system present, this transition is forbidden because there is no matter in any form present (except the electromagnetic waves themselves) to receive the energy released from the wave. This is where the constancy of the photon number results from.

This general law of forbiddenness may still have some exceptions. If on a certain specific condition involving transition (6.2) no change in energy takes place, the process may also proceed in the vacuum since the reason of forbiddenness, based on the energy balance, is absent. Since such a transition is not caused by a physical system different from the waves, the transition may be called a spontaneous one. A characteristic feature of spontaneous transitions is the decrease of the photon number without any change in the wave energy. The moment $t=t_1$ of the spontaneous transition must then be considered as another moment of excitation when the wave consisting of n photons is replaced by that consisting of $n-1$ photons.

A spontaneous transition is a discrete process and it can take place only at discrete intervals of time. Let our task be to obtain a formula for the calculation of this interval.

In the transition the quantum number n changes discretely according to (6.2). Accordingly, the wave function of the oscillator must also change discretely. However, a change in the wave function is restricted by two conditions: the energy calculated by means of wave functions must remain constant at a jump, whereas the quantum number n must decrease by unity.

The wave function of the oscillator can be represented according to (5.11) in the form

$$\Phi=C_n\cdot\varphi_n(q)\cdot\exp\left[-i\omega\left(n+\frac{1}{2}\right)\exp[-4\pi\sigma(t-t_0)]\cdot\tau\right], \quad (6.3)$$

in which case the following substitutions have been made in (5.11):

$$E_n^{(0)}=\hbar\omega\left(n+\frac{1}{2}\right)$$

and

$$\frac{8\pi^2\sigma}{\varkappa}m=t-t_0.$$

These substitutions have already been used above, in which case t_0 is the excitation moment of the oscillator.

Let the moment of the spontaneous transition be $t=t_1$. During the short time interval Δt prior to the spontaneous transition, i. e. at the moment

$$t=t_1-\Delta t$$

the wave function of the oscillator according to (6.3) is

$$\Phi=C_n\cdot\varphi_n(q)\cdot\exp\left[-i\omega\left(n+\frac{1}{2}\right)\exp[-4\pi\sigma(t_1-t_0-\Delta t)]\cdot\tau\right]. \tag{6.4}$$

After the lapse of the time interval Δt since the jump, i. e. at the moment

$$t=t_1+\Delta t$$

the wave function is

$$\Phi=C_{n-1}\cdot\varphi_n(q)\cdot\exp\left[-i\omega\left(n-\frac{1}{2}\right)\cdot\exp[-4\pi\sigma\Delta t]\cdot\tau\right]. \tag{6.5}$$

When writing this formula, it was taken into account that in the spontaneous transition the quantum number n decreases by unity and that the moment of the transition $t=t_1$ is a new moment of excitation of the oscillator. Therefore t_0 in (6.3) denoting the initial moment of excitation must be replaced by t_1 denoting the new moment of excitation, i. e.

$$t_0\to t_1.$$

The energy of the oscillator immediately before the transition can be calculated by means of the wave function (6.4)

$$E_{t<t_1}=-\int\Phi^*\frac{\hbar}{i}\frac{\partial}{\partial\tau}\Phi\cdot dq=$$

$$=\hbar\omega\left(n+\frac{1}{2}\right)\exp[-4\pi\sigma(t_0-t_1-\Delta t)]. \tag{6.6}$$

Likewise the calculable energy immediately following the spontaneous transition is

$$E_{t>t_1}=\hbar\omega\left(n-\frac{1}{2}\right)\cdot\exp[-4\pi\sigma\Delta t]. \tag{6.7}$$

Energy does not undergo any changes in the spontaneous transition. Hence

$$E_{t<t_1} = E_{t>t_1}$$

if $\Delta t \to 0$. From (6.6) and (6.7) we obtain the following:

$$t_1 - t_0 = \frac{1}{4\pi\sigma} \ln \frac{2n+1}{2n-1}, \qquad (6.8)$$

which is the formula we have been aiming at.

Formula (6.8) helped to interpret the multiplicity of the redshift of quasars [2].

Formula (6.8) has been derived on the assumption that spontaneous transitions really take place; it does not prove, however, their occurrence. As far as the latter is concerned, a respective motivation has been given in the author's previous work [2]. As the main motivation one should regard the consideration following from the energy balance. The spontaneous transitions without any accompanying energy redistribution may take place with a certain probability different from zero but presumably less than unity. This fact, however, is typical of the rules of quantum physics.

Summary

The leading idea of this investigation is the hypothesis according to which the vacuum possesses a number of physical properties, including the ability to absorb the electromagnetic radiation passing through it. In comparison with real matter, however, the radiation absorption effect of the vacuum is negligible. But light, coming to the observer from cosmological objects, has been affected by the vacuum for millions of years. Here the vacuum must have exercised an influence on radiation which may manifest itself, for example, in the well-known redshift of the spectral lines of cosmological objects. This idea provides the basis for the creation of the theory of cosmological quantum electrodynamics.

According to the theory of cosmological quantum electrodynamics, the state of a monochromatic wave is determined by two numerical quantities — the wave photon number n and the photon frequency $\bar{\omega}$. The energy of the whole wave is

$$E=\hbar\bar{\omega}\left(n+\frac{1}{2}\right),$$

in which case this formula is formally entirely similar to that of the quantum theory of electromagnetic radiation, which does not take into account the physical properties of the vacuum. In the traditional quantum theory of radiation the photon number as well as frequency are constant. According to the theory of cosmological electrodynamics, however, both these quantities change in time. The formulae for the establishment of these changes are given in the present paper.

One of the most characteristic features of this theory is the change of a monochromatic wave in the vacuum. At the same time, these changes are either continuous or discrete. In a continuous transition the wave energy changes while the photon number remains unchanged. In discrete transitions, however, it is the photon number that decreases, whereas the wave energy remains unchanged.

The continuous decrease of photon energy coupled with a decrease in frequency gives the well-known redshift formula. As is commonly known, the latter is already derived from general cosmolog'cal considerations without making use of the Maxwell equations or their quantization. On the contrary, the discrete change of the photon number is a typical quantum effect and it cannot be derived without applying the rules of quantum physics. This fact should justify the attempt to suggest a theory of cosmological quantum electrodynamics.

Due to the discrete changes in the photon number which take place at discrete moments and which extinguish the redshift, one can expect to find multiplicity of the redshift quantity z of the spectral lines of cosmological objects. As we know, this has been observed in various absorption lines, especially in those which occur in the spectra of remote quasars. The multiplicity of the redshift quantity is one of the most decisive facts estimating the validity of the theory of cosmological electrodynamics presented in this work.

The problem of the relations between the theory of cosmological quantum electrodynamics and the generally acknowledged cosmological theories has been treated as a separate question in the present paper. The cosmological theories have been developed to perfection on the principles of the general theory of relativity. We are of the opinion that these two groups of theories are not antagonistic, excluding each other. Quantum physics does not eliminate macrophysics and something of the kind could be expected after the theory of cosmological quantum electrodynamics developed in this paper has justified itself.

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